7. Double integrals.

1. Evaluate double integrals of given functions f over the rectangles indicated below.

   (a) $f(x,y) = xye^{2xy^2}$, $D = [0,2] \times [1,2]$.

   (b) $f(x,y) = \dfrac{1}{(2x-y)^3} + \dfrac{1}{2x-y}$, $D = [1,2] \times [-1,0]$.

   (c) $f(x,t) = \dfrac{1}{\sqrt{(x-t^2)^3}}$, $D = \left\{(x,t) : 0 \le x \le 4,\ 0 \le t \le \dfrac{1}{2}\right\}$.

   (d) $f(r,\varphi) = r(r^2\cos^2\varphi + r\sin(2\varphi) + \tan\varphi)$, $D = \left\{(r,\varphi) : 1 \le r \le 3,\ 0 \le \varphi \le \frac{\pi}{3}\right\}$.

2. Let g be a function that is integrable over $[a,b]$ and let h be a function that is integrable over $[c,d]$. Show that if $f = f(x,y)$ can be represented as their product, that is, $f(x,y) = g(x) \cdot h(y)$, then over the rectangle $D = [a,b] \times [c,d]$ the integral of f may be decomposed into a product of two definite integrals, that is,

$$\iint_D f(x,y)dxdy = \int_a^b g(x)dx \cdot \int_c^d h(y)dy.$$

   Use this formula to evaluate $\displaystyle\iint_D e^{x^2+y^3}\sin x dxdy$ na $D = [-\pi,\pi] \times [1,2]$.

3. Consider the sets D that are bounded by the graphs of the curves given below. Represent $\displaystyle\iint_D f(x,y)dxdy$ as an iterate integral or a sum of iterated integrals.

   Use both orders of integration and, hence, give both results.

   (a) $y = \dfrac{1}{2}(x+1)$, $x = 1$, $y = 2$.

   (b) $y = \sqrt{6-x^2}$, $y = x^2$.

   (c) $y = \sqrt{x^2-4}$, $y = 0$, $y = 1$.

   (d) $y = 2x-1$, $y = -5-x$, $y = 2$.

4. Evaluate double integrals of given functions f over the sets indicated below.

   (a) $f(x,y) = xy^{10}$, D is bounded by the graphs of the curves $y = x$ and $y = 2-x^2$.

   (b) $f(x,y) = 2^{x^3}y$, D is a triangle with vertices $(0,0)$, $(1,-1)$ and $(1,0)$.

   (c) $f(x,y) = \dfrac{1}{y}e^{\frac{x}{y^2}}$, $D = \left\{(x,y) : \dfrac{1}{4} \le x \le 2,\ \sqrt[3]{\dfrac{1}{2}x} \le y \le 1\right\}$.

   (d) $f(x,y) = x+2y$, D is bounded by the graphs of the curves $x = 0$, $x = \dfrac{\pi}{3}$, $y = \cos(2x)$ i $y = -2$.

   (e) $f(x,y) = \dfrac{x}{(xy+3)^2}$, D is bounded by the X-axis and the graphs of the lines $x = 3$ i $y = x$.

   (f) $f(x,y) = x\sqrt[3]{y}$, $D = \left\{(x,y) : 1 \le y \le \sqrt{4-x^2}\right\}$.

5. Using polar coordinates calculate double integrals of given functions f over the sets indicated below.

   (a) $f(x,y) = \frac{x}{y^4}$, $D = \{(x,y) : 1 \le x^2 + y^2 \le 3,\ 0 \le y \le -x\}$.

   (b) $f(x,y) = \frac{3}{y}$, $D = \{(x,y) : 1 \le x^2 + y^2 \le 2y\}$.

   (c) $f(x,y) = y$, $D = \{(x,y) : x^2 + y^2 \le 2x,\ 0 \le y \le x\}$.

   (d) $f(x,y) = x^4 + y^4$, D is bounded by the graphs of the curves $y = \frac{1}{2}(x + |x|)$ and $y = \sqrt{4 - x^2}$.

   (e) $f(x,y) = 3x + 2y$, D is bounded by the graphs of the curves $y = 3 + \sqrt{11 - 4x - x^2}$, $x = -2$ and $y = x + 5$.

   (f) $f(x,y) = \frac{1}{\sqrt{(x^2 + y^2)^3}}$, $D = \left\{(x,y) : 1 \le y \le \sqrt{2 - x^2}\right\}$.

6. In polar coordinates the image of a set $D \subset \mathbf{R}^2$ is a rectangle. What kind of set may D be in cartesian coordinates? Describe its shape, position and parameters.

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