

Exponential and logarithmic equations and inequalities

1. Solve, exactly,

(a) $4^x - 7 \cdot 2^x + 10 = 0$.

(b) $2e^x - 21e^{-x} - 1 = 0$.

(c) $25^x + 7 \cdot 5^x + 12 = 0$.

(d) $e^{3x} - 3e^x + 2 = 0$.

(e) $8^x - 2 \cdot 4^x - 9 = 0$.

2. Find the inverse functions to

(a) $y = 2e^{2x} + 3e^x - 7$.

(b) $y = \frac{e^x + e^{-x}}{2}$.

(c) $y = 9^x + 3^x - 2$.

3. Find, exactly, the range of the functions below.

(a) $y = 2e^{2x} - 7e^x - 21$.

(b) $y = 25^x + 7 \cdot 5^x + 12$.

(c) $y = 2e^x + e^{-x}$.

4. Solve, exactly, the inequalities below.

(a) $4e^{2x} - 13e^x + 3 \geq 0$.

(b) $2 - 10^x - 15 \cdot 0.1^x > 0$.

(c) $9^x - 2 \cdot 3^x - 8 \geq 0$.

(d) $-3e^{2x} - e^x + 2 \geq 0$.

(e) $4^x + 8 \cdot 2^x + 7 > 0$.

(f) $2 \cdot 9^x + 17 \cdot 3^x + 30 \leq 0$.

(g) $e^{2x} - 6e^x + 9 > 0$.

(h) $4 \cdot 9^x + 20 \cdot 3^x + 25 > 0$.

(i) $e^{2x} - 3e^x + 4 \geq 0$.

(j) $-6 \cdot 5^x - 5 \cdot 0.2^x + 1 > 0$.

(k) $8^x - 2 \cdot 4^x - 5 \cdot 2^x + 6 \geq 0$.

(l) $2e^{3x} - 15e^{2x} + 37e^x - 30 > 0$.

(m) $-27^x + 5 \cdot 9^x - 8 \cdot 3^x - 4 < 0$.

(n) $e^{4x} - 3e^{3x} - 3e^{2x} + 11e^x - 6 < 0$.

(o) $16^x - 5 \cdot 8^x + 5 \cdot 4^x + 5 \cdot 2^x - 6 \leq 0$.

5. Solve, exactly,

(a) $(\log x)^2 - 7 \log x + 10 = 0$.

(b) $2(\log_2 x)^2 - \log_2 x - 21 = 0$.

(c) $(\ln x)^2 + 7 \ln x + 12 = 0$.

(d) $(\ln x)^3 - 3 \ln x + 2 = 0$.

(e) $(\log_3 x)^3 - 2(\log_3 x)^2 - 9 = 0$.

6. Find the inverse functions to

(a) $y = \ln(x + 1) - \ln x$.

(b) $y = \log(x + 1) + \log x$.

(c) $y = \log_3(x^2 + 1) - 2 \log_3 x$.

7. Find, exactly, the range of the functions below.

(a) $y = 2(\log x)^2 - 7 \log x - 21$.

(b) $y = (\log_2 x)^2 + 7 \log_2 x + 12$.

(c) $y = 2 \ln x + \frac{1}{\ln x}$.

8. Solve, exactly, the inequalities below (compare it to task 4).

(a) $4(\log_5 x)^2 - 13 \log_5 x + 3 \geq 0$.

(b) $-(\log x)^2 + 2 \log x - 15 > 0$.

(c) $(\ln x)^2 - 2 \ln x - 8 \geq 0$.

(d) $-3(\log_2 x)^2 - \log_2 x + 2 \geq 0$.

(e) $(\log_{0.8} x)^2 + 8 \log_{0.8} x + 7 > 0$.

(f) $2(\log_3 x)^3 + 17 \log_3 x + 30 \leq 0$.

(g) $(\ln x)^2 - 6 \ln x + 9 > 0$.

(h) $4(\log x)^2 + 20 \log x + 25 > 0$.

(i) $(\log_6 x)^2 - 3 \log_6 x + 4 \geq 0$.

(j) $-6(\log_4 x)^2 - 5 \log_4 x + 1 > 0$.

(k) $(\log_2 x)^3 - 2(\log_2 x)^2 - 5 \log_2 x + 6 \geq 0$.

(l) $2(\ln x)^3 - 15(\ln x)^2 + 37 \ln x - 30 > 0$.

(m) $-(\log x)^3 + 5(\log x)^2 - 8 \log x - 4 < 0$.

(n) $(\log_6 x)^4 - 3(\log_6 x)^3 - 3(\log_6 x)^2 + 11 \log_6 x - 6 < 0$.

(o) $(\log_3 x)^4 - 5(\log_3 x)^3 + 5(\log_3 x)^2 + 5 \log_3 x - 6 \leq 0$.

9. Solve, exactly,

(a) $\log_2(x + 1) - \log_2(x - 1) \leq 4$.

(b) $\log(3x^2 + 28) - \log(3x - 2) < 1$.

(c) $\log_6(x + 5) + \log_6 x \geq 2$.

(d) $\log_3(x - 2) + \log_3(x - 4) > 2$.

(e) $\ln(x + 1) - \ln x < 4$.