

List 0. Binomial expansion and trigonometric equations.

1. Expand and simplify

(a) $(2x + 3)^4$,

(b) $\left(3x - \frac{1}{3x^2}\right)^5$,

(c) $\left(\sqrt{y} + \frac{2}{y}\right)^6$,

(d) $(a - 1)^8$.

2. In the expansion of $\left(x^3 - \frac{2}{x}\right)^{36}$ find

(a) the coefficient of x^{16} ,

(b) the coefficient of $\frac{1}{x^{12}}$,

(c) the constant term (independent of x).

3. In the expansion of $(1 + 3x)^n$ the coefficient of x^2 is 90. Find n .

4. In the expansion of $(1 + ax)^n$ the first three terms are 1, $24x$ and $252x^2$. Find a and n .

5. Find an approximation of 1.02^{38} by taking the sum of the first four terms in the expansion of $(1 + 0.02)^{38}$. Find the percentage error of this approximation.

6. Using the expansion of $(1 + x)^n$ show that for all positive natural n

(a) $8^n + 6$ is divisible by 7,

(b) $4^n + 2$ is divisible by 6,

(c) $6^n + 3 \cdot 11^n + 1$ is divisible by 5,

(d) $4^n + 6n - 1$ is divisible by 9,

(e) $5^n + 12n + 15$ is divisible by 16,

(f) $3^n - 2n^2 + 7$ is divisible by 8.

7. Solve, exactly, the equations below.

(a) $\sin x = \frac{\sqrt{3}}{2}$ in $[0, 2\pi]$.

(b) $\cos x = -\frac{1}{2}$ in $[-\pi, 2\pi]$.

(c) $\tan x = 1$ in $[0, 3\pi]$.

(d) $\sin x = 1$ in $[-120^\circ, 240^\circ]$.

(e) $\cos x = \frac{\sqrt{2}}{2}$ in $[-300^\circ, 30^\circ]$.

(f) $\tan x = -\sqrt{3}$ in $[-180^\circ, 360^\circ]$.

(g) $\sin(2x) = \frac{1}{2}$ in $[0, 2\pi]$.

(h) $\cos\left(\frac{1}{2}x\right) = -1$ in $[-2\pi, 3\pi]$.

(i) $\sin\left(\frac{2}{3}x\right) = -\frac{1}{2}$ in $[-180^\circ, 180^\circ]$.

(j) $\cos(3x) = \frac{\sqrt{2}}{2}$ in $[0^\circ, 360^\circ]$.