Math-algebra. Linear equations

1. Let $a, b, \alpha \in \mathbf{R}$ be fixed parameters. Using Cramer's rule solve the systems below.

a) $\begin{cases} 6x+25y=-1 \\ -x+y=2 \end{cases}$ b) $\begin{cases} x\cos\alpha - y\sin\alpha = a \\ x\sin\alpha + y\cos\alpha = b \end{cases}$ c) $\begin{cases} \dfrac{1-a^2}{1+a^2}x + \dfrac{2a}{1+a^2}y = 1 \\ \dfrac{2a}{1+a^2}x - \dfrac{1-a^2}{1+a^2}y = a \end{cases}$

d) $\begin{cases} \dfrac{1}{1+a^2}x + \dfrac{a}{1+a^2}y = -a \\ \dfrac{a}{1+a^2}x + \dfrac{a^2}{1+a^2}y = 1 \end{cases}$ e) $\begin{cases} x+2y+z=-1 \\ x+y+3z=2 \\ x+3y+8z=1 \end{cases}$ f) $\begin{cases} x+2y+z=-1 \\ x+y+3z=2 \\ 2x+3y+4z=6 \end{cases}$

2. Solve, when possible, the systems from the previous task using the inverse matrix method.

3. Solve the systems below using the method of elimination.

a) $\begin{cases} x+2y+z=-1 \\ x+y+3z=2 \\ x+3y+8z=1 \end{cases}$ b) $\begin{cases} x+2y+z=-1 \\ x+y+3z=2 \\ 2x+3y+4z=6 \end{cases}$ c) $\begin{cases} x+2y+z=-1 \\ x+y+3z=2 \\ 2x+3y+4z=1 \end{cases}$

d) $\begin{cases} x+2y+z=-1 \\ x+y+3z=2 \end{cases}$ e) $\begin{cases} x-2y+z=-4 \\ x+y+z=1 \\ 2x-3y+5z=10 \\ 5x-6y+18z=19 \end{cases}$ f) $\begin{cases} x+2y+z+t=7 \\ 2x-y-z+4t=2 \\ 5x+5y+2z+7t=1 \end{cases}$

g) $\begin{cases} x+2y+z+t=7 \\ 2x-y-z+4t=-20 \\ 5x+5y+2z+7t=1 \end{cases}$ h) $\begin{cases} x-2y+z-t=-4 \\ 2x-y-z+t=1 \\ x+y+2z-t=5 \\ x+y-z+t=4 \end{cases}$ i) $\begin{cases} x+2y+3z+t=1 \\ 2x+4y-z+2t=2 \\ 3x+6y+10z+3t=3 \\ x+y+z+t=0 \end{cases}$

j) $\begin{cases} 3x+y-2t=1 \\ 5x+2y+2z-t=5 \\ x-y-2t=-5 \\ 5x+y+z-3t=0 \\ -7x-3y+z+5t=-4 \end{cases}$ k) $\begin{cases} x-y+z-2s+t=0 \\ 3x+4y-z+s+3t=1 \\ x-8y+5z-9s+t=-1 \end{cases}$ l) $\begin{cases} x-3y+z-2s+t=-5 \\ 2x-6y-4s+t=-10 \\ 2z+t=0 \\ -2x+6y+2z+4s=10 \\ -2x+6y+4z+4s+t=10 \\ -x+3y+z+2s=5 \end{cases}$

4. Solve the system $\begin{cases} x+2y+z=-1 \\ x-y+3z=0 \\ 2x+3y-z=1 \end{cases}$ using

a) the inverse matrix method,

b) Cramer's rule,

c) the elimination.

5. Find all $p \in \mathbf{R}$ for which the systems below have unique solutions. For other values of p solve the systems.

a) $\begin{cases} x+2y+z=-1 \\ y+3z=2 \\ x+3y+pz=0 \end{cases}$ b) $\begin{cases} (p+5)x+2y+4z=2 \\ 4x+py+2z=2 \\ 3x+y+2z=1 \end{cases}$ c) $\begin{cases} x+4y-2z=-p \\ 3x+5y-pz=3 \\ px+3py+z=p \end{cases}$

6. Determine for which values of $p \in \mathbf{R}$ the systems below are solvable and solve them.

a) $\begin{cases} x+y+z=1 \\ 2x-y+3z=5 \\ x-2y+2z=p \end{cases}$ b) $\begin{cases} x+2y+z+t=3 \\ 2x+3y-5z+t=p \\ -x-y+6z=1 \end{cases}$ c) $\begin{cases} x+2y-3z=2 \\ -3x+y-z=p \\ -2x+3y-4z=p^2 \end{cases}$

7. Consider a system of linear equations written as $AX = B$. Let $A \neq \mathbf{0}$ be square. A popular error is the following statement. If all the determinants related to the system (that is, $\det A$ and the determinants associated with the variables) are equal to 0 then the system has infinitely solutions.

   Give an example of a system that disproves this statement.

8. Consider a system of linear equations written as $AX = B$. Let $A \neq \mathbf{0}$ be square and of dimension 2. Using Kronecker-Capelli theorem prove that in this special case the statement from the previous task is true. That is, if all three determinants related to the system are 0 then there are infinitely many solutions.

9. Prove that if a system of linear equations has more variables than equations then is cannot have a unique solution.

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