Math-algebra. Lines and planes.

1. Find the cartesian and the parametric form of a line that

   (a) joins the origin $(0, 0)$ to $P = (1, 2)$,

   (b) joins $A = (2, 3)$ to $B = (-1, 5)$,

   (c) passes through $P = (3, 2)$ and is parallel to the line $y = 2x$,

   (d) passes through $P = (3, 2)$ and is perpendicular to the line $y = 3x - 1$.

2. Find the distance from the line $y = 2x + 1$ to

   (a) the point $P = (3, 2)$,

   (b) the line $-4x + 2y + 5 = 0$,

   (c) the line $8x + 9y + 7 = 0$.

3. Find all lines that

   (a) pass through the origin and their distance to the point $P = (1, 2)$ is equal to 1,

   (b) their distance to the line $y = 2x + 1$ is equal to 4.

4. Find the cartesian and the parametric form of a plane that

   (a) is parallel to $\vec{u} = [1, 2, 3]$ and to $\vec{v} = [0, 5, 2]$,

   (b) passes through $A = (2, 3, 5)$, $B = (1, 7, 0)$ and $C = (4, 8, 1)$,

   (c) passes through the origin and is perpendicular to the line that joins $A = (1, 2, 3)$ to $B = (7, 2, 0)$,

   (d) passes through $P = (1, -1, 3)$ and includes the Z-axis.

5. Find the cartesian and the parametric form of a line that

   (a) joins the origin $(0, 0, 0)$ to $P = (1, 2, 4)$,

   (b) joins $A = (1, 2, 3)$ to $B = (-1, 0, 5)$,

   (c) passes through $P = (7, 3, 2)$ and is perpendicular to the YZ plane,

   (d) is the common part of the planes $\Pi_1 : x - y + z + 1 = 0$ and $\Pi_2 : 2x - y + 3z = 0$,

   (e) passes through $P = (3, 2, -1)$ and is parallel to the Z-axis.

6. An object is moving in 3 dimensions with a constant velocity $\vec{v} = [2, -2, 1]$, where each coordinate of $\vec{v}$ is given in m/s. Let t represent time, measured in seconds. When $t = 0$ the object is at $P_0 = (1, 6, -7)$, where each coordinate is given in meters.

   (a) Find, in parametric form, the trajectory of the object as a function of time t.

   (b) Find the position of the object

   - at $t = 5$,
   - 2 seconds before it reached P_0 .

   (c) Find the position of the object when it meets the XZ plane.

   (d) Investigate whether the object reaches the points $A = (17, -10, 1)$ and $B = (11, -4, -1)$.

   (e) Find the speed of this motion and the distance the object covers within 4 seconds.

7. Describe mutual position of the planes below.

(a) $\Pi_1 : 3x - 2y + 3z + 1 = 0$ and $\Pi_2 : x - y - z - 7 = 0$,

(b) $\Pi_1 : 6x + 2y - 4z - 10 = 0$ and $\Pi_2 : -3x - y + 2z + 5 = 0$,

(c) $\Pi_1 : x - 2y + 3z = 0$ and $\Pi_2 : 0.1x - 0.2y + 0.3z + 1 = 0$,

(d) $\Pi_1 : x - 2y - 3z - 4 = 0$ and $\Pi_2 : \begin{cases} x = 3 - s + t \\ y = 2 + s - t \\ z = 1 + 2s + t \end{cases}, s, t \in \mathbf{R}$,

(e) $\Pi_1 : x - 2y - 3z - 4 = 0$ and $\Pi_2 : \begin{cases} x = 1 + s + t \\ y = 2s - t \\ z = -1 - s + t \end{cases}, s, t \in \mathbf{R}$,

(f) $\Pi_1 : x - 2y - 3z - 4 = 0$ and $\Pi_2 : \begin{cases} x = 2 - s - t \\ y = -4 - 2s + t \\ z = 7 + s - t \end{cases}, s, t \in \mathbf{R}$,

(g) $\Pi_1 : \begin{cases} x = 2 + s + t \\ y = 4 - s \\ z = 5s + 3t \end{cases}, s, t \in \mathbf{R}$, and $\Pi_2 : \begin{cases} x = 2 + a + b \\ y = 1 - a - b \\ z = 2 - 3a + 2b \end{cases}, a, b \in \mathbf{R}$,

(h) $\Pi_1 : \begin{cases} x = s + t \\ y = s + 3t \\ z = -s - 2t \end{cases}, s, t \in \mathbf{R}$, and $\Pi_2 : \begin{cases} x = a - 3b \\ y = -a - b \\ z = 2b \end{cases}, a, b \in \mathbf{R}$,

(i) $\Pi_1 : \begin{cases} x = 2 + 3s + 5t \\ y = 4 + s + t \\ z = -1 - 2s - 3t \end{cases}, s, t \in \mathbf{R}$, and $\Pi_2 : \begin{cases} x = 5 - 4a + b \\ y = 1 - 2a - 3b \\ z = 3a + b \end{cases}, a, b \in \mathbf{R}$,

(j) $\Pi_1 : Ax + By + Cz + D = 0$ and $\Pi_2 : 2Ax + 2By + 2Cz + E = 0$,
where $A, B, C, D, E \in \mathbf{R}$ and $A \neq 0 \vee B \neq 0 \vee C \neq 0$,

(k) $\Pi_1 : x + By + Cz + D = 0$ and $\Pi_2 : -2x + 3y - z + 1 = 0$,
where $B, C, D \in \mathbf{R}$,

(l) $\Pi_1 : Ax + z + 5 = 0$ and $\Pi_2 : x + By - z + 1 = 0$,
where $A, B, \in \mathbf{R}$,

(m) $\Pi_1 : Ax + By + D = 0$ and $\Pi_2 : x - z + E = 0$,
where $A, B, D, E \in \mathbf{R}$ and $A \neq 0 \vee B \neq 0$.

8. Describe mutual position of the lines below. When they intersect at one point find its coordinates.

(a) $L_1 : \begin{cases} x = 1 - s \\ y = 2 + s \\ z = 3 + 2s \end{cases}, s \in \mathbf{R}$, and $L_2 : \begin{cases} x = 2t \\ y = 3 - 2t \\ z = 1 - 4t \end{cases}, t \in \mathbf{R}$,

(b) $L_1 : \dfrac{x-2}{3} = \dfrac{y-3}{-2} = \dfrac{z+1}{5}$ and $L_2 : \dfrac{x-5}{-3} = \dfrac{y-1}{2} = \dfrac{z-4}{-5}$,

(c) $L_1 : \begin{cases} x = t \\ y = 2 - t \\ z = 1 + t \end{cases}, t \in \mathbf{R}$, and $L_2 : \begin{cases} x + y + z - 5 = 0 \\ 2y - z + 3 = 0 \end{cases}$,

(d) $L_1 : \begin{cases} x = 2 - s \\ y = 3 + s \\ z = 4 \end{cases}, s \in \mathbf{R}$, and $L_2 : \begin{cases} x = 3 + t \\ y = 2 + t \\ z = -3 + t \end{cases}, t \in \mathbf{R}$,

(e) $L_1 : x = y = 2z$ and $L_2 : \begin{cases} x = 1 + 2t \\ y = -1 + 2t \\ z = t \end{cases}, t \in \mathbf{R}$,

(f) $L_1 : \begin{cases} x = 3 - t \\ y = 2 + t \\ z = 1 + 2t \end{cases}, t \in \mathbf{R}$, and $L_2 : \begin{cases} x - y + z - 2 = 0 \\ x + y - 5 = 0 \end{cases}$,

(g) $L_1 : \begin{cases} x = 2 + s \\ y = 3 - s \\ z = 4s \end{cases}, s \in \mathbf{R}$, and $L_2 : \begin{cases} x = 1 + 2t \\ y = 1 + t \\ z = 5 - t \end{cases}, t \in \mathbf{R}$,

(h) $L_1 : \begin{cases} x + 2y - 3z + 1 = 0 \\ x + y + z - 1 = 0 \end{cases}, t \in \mathbf{R}$, and $L_2 : \begin{cases} x - y + 3 = 0 \\ y + 2z + 1 = 0 \end{cases}$,

(i) $L_1 : \begin{cases} x = 1 - s \\ y = 2 + s \\ z = 3 + as \end{cases}, s \in \mathbf{R}$, and $L_2 : \begin{cases} x = 2t \\ y = 3 - 2t \\ z = 1 - 4t \end{cases}, t \in \mathbf{R}$, and $a \in \mathbf{R}$,

(j) $L_1 : \begin{cases} x = 3 - t \\ y = 2 + t \\ z = a + 2t \end{cases}, t \in \mathbf{R}$, and $L_2 : \begin{cases} x - y + z - 2 = 0 \\ x + y - 5 = 0 \end{cases}$, and $a \in \mathbf{R}$,

(k) $L_1 : \begin{cases} x = 2 - s \\ y = 3 + s \\ z = a \end{cases}, s \in \mathbf{R}$, and $L_2 : \begin{cases} x = 3 + t \\ y = 2 + t \\ z = -3 + t \end{cases}, t \in \mathbf{R}$, and $a \in \mathbf{R}$,

9. Describe mutual position of the lines and the planes below. When they have common points find their coordinates.

(a) $L : \begin{cases} x = 1 - t \\ y = 2 + t \\ z = 3t \end{cases}, t \in \mathbf{R}$, and $\Pi : 3x + 2y - 5z + 1 = 0$,

(b) $L : \begin{cases} x = 2t \\ y = 1 - t \\ z = 3 + t \end{cases}, t \in \mathbf{R}$, and $\Pi : x + y - z - 2 = 0$,

(c) $L : \begin{cases} x = -3 + 2t \\ y = 1 - t \\ z = 3 + t \end{cases}, t \in \mathbf{R}$, and $\Pi : -x - y + z + 9 = 0$,

(d) $L : \begin{cases} x + 2y - z + 1 = 0 \\ 2x + y - 3 = 0 \end{cases}$ and $\Pi : 2x + 3y - z = 0$,

(e) $L : \begin{cases} 2x - 3y + z = 0 \\ 3x - 2y + 2z = 0 \end{cases}$ and $\Pi : x + y + z + 1 = 0$,

(f) $L : \begin{cases} 2x - 3y + z + 1 = 0 \\ 3x - 2y + 2z + 2 = 0 \end{cases}$ and $\Pi : x + y + z + 1 = 0$,

(g) $L : \begin{cases} x = 1 - t \\ y = 2 + t \\ z = at \end{cases}, t \in \mathbf{R}, a \in \mathbf{R}$, and $\Pi : 3x + 2y - 5z + 1 = 0$,

(h) $L : \begin{cases} x = a + 2t \\ y = 1 - t \\ z = 3 + t \end{cases}, t \in \mathbf{R}, a \in \mathbf{R}$, and $\Pi : x + y - z - 2 = 0$,

(i) $L : \begin{cases} 2x - 3y + z + 1 = 0 \\ 3x - 2y + 2z + 2 = 0 \end{cases}$ and $\Pi : ax + y + z + 1 = 0$, $a \in \mathbf{R}$.

10. Find the distance between

(a) $\Pi_1 : x - y + 2z + 1 = 0$ and $\Pi_2 : 2x - 2y + 4z + 7 = 0$,

(b) $\Pi_1 : x + 2y + 3z = 0$ and $\Pi_2 : \begin{cases} x = 1 - s + t \\ y = 2 - s \\ z = 2s + 3t \end{cases}, s, t \in \mathbf{R}$,

(c) $\Pi : x + y + z + 1 = 0$ and $L : \begin{cases} 2x - 3y + z = 0 \\ 3x - 2y + 2z = 0 \end{cases}$,

(d) $\Pi : x + y - z - 2 = 0$ and $L : \begin{cases} x = 2t \\ y = 1 - t \\ z = 3 + t \end{cases}, t \in \mathbf{R}$,

(e) $\Pi : 3x - 2y + z + 1 = 0$ and $L : \begin{cases} x = 2 - t \\ y = 3 + t \\ z = 4t \end{cases}, t \in \mathbf{R}$.

11. Consider two planes – $\Pi_1 : 2x + 3z + 6z + 1 = 0$ and $\Pi_2 : Ax + By + z + D = 0$, where $A, B, D \in \mathbf{R}$. Find the values of A, B, D for which the distance between the planes is equal to 2.

12. Consider the plane $\Pi : x - y + 2z + 1 = 0$ and the line $L : \begin{cases} x = 1 + t \\ y = 2 - t \\ z = a + bt \end{cases}, t \in \mathbf{R}$, and $a, b \in \mathbf{R}$.

Find the values of a and b for which

(a) L is included in Π,

(b) the distance from L to Π is equal to 1,

(c) the only common point of L and Π is $P = (2, 1, -1)$.

13. Find the distance between

(a) $P = (2, 1, -1)$ and $L : \begin{cases} x = 3 - t \\ y = 2t \\ z = 1 + t \end{cases}, t \in \mathbf{R}$.

(b) $L_1 : \begin{cases} x = 1 - s \\ y = 2 + s \\ z = 3 + 2s \end{cases}, s \in \mathbf{R}$, and $L_2 : \begin{cases} x = 2t \\ y = 3 - 2t \\ z = 1 - 4t \end{cases}, t \in \mathbf{R}$,

(c) $L_1 : \begin{cases} x = 2 - s \\ y = 3 + s \\ z = 4 \end{cases}, s \in \mathbf{R}$, and $L_2 : \begin{cases} x = 3 + t \\ y = 2 + t \\ z = -3 + t \end{cases}, t \in \mathbf{R}$,

(d) $L_1 : x = y = 2z$ and $L_2 : \begin{cases} x = 1 + 2t \\ y = -1 + 2t \\ z = t \end{cases}, t \in \mathbf{R}$,

(e) $L_1 : \begin{cases} x + 2y - 3z + 1 = 0 \\ x + y + z - 1 = 0 \end{cases}, t \in \mathbf{R}$, and $L_2 : \begin{cases} x - y + 3 = 0 \\ y + 2z + 1 = 0 \end{cases}$,

14. Find the projection of $P = (1, 2, 5)$ onto

(a) $\Pi : x + 2y - 2z + 1 = 0$,

(b) $\Pi : \begin{cases} x = 1 - s + t \\ y = 2 + t \\ z = 3 - s - t \end{cases}, s, t \in \mathbf{R}$,

(c) $L: \begin{cases} x = t \\ y = 2t \\ z = 3t \end{cases}, t \in \mathbf{R},$

(d) $L: \begin{cases} x + y + z + 1 = 0 \\ x - y + 2z = 0 \end{cases}.$

15. Find the exact value of the angle between

(a) $L_1 : x = y = 2z$ and $L_2 : \begin{cases} x = 1 + 2t \\ y = -1 + 2t \\ z = t \end{cases}, t \in \mathbf{R},$

(b) $L_1 : \begin{cases} x = 2 - s \\ y = 3 + s \\ z = 4 \end{cases}, s \in \mathbf{R},$ and $L_2 : \begin{cases} x = 3 + t \\ y = 2 + t \\ z = -3 + t \end{cases}, t \in \mathbf{R},$

(c) $L_1 : \begin{cases} x = t \\ y = 2 - t \\ z = 1 + t \end{cases}, t \in \mathbf{R},$ and $L_2 : \begin{cases} x + y + z - 5 = 0 \\ 2y - z + 3 = 0 \end{cases},$

(d) $\Pi_1 : 3x - 2y + 3z + 1 = 0$ and $\Pi_2 : x - y - z - 7 = 0,$

(e) $\Pi_1 : x - 2y + 3z = 0$ and $\Pi_2 : 0.1x - 0.2y + 0.3z + 1 = 0,$

(f) $\Pi_1 : x - 2y - 3z - 4 = 0$ and $\Pi_2 : \begin{cases} x = 3 - s + t \\ y = 2 + s - t \\ z = 1 + 2s + t \end{cases}, s, t \in \mathbf{R},$

(g) $L : \begin{cases} x = 1 - t \\ y = 2 + t \\ z = 3t \end{cases}, t \in \mathbf{R},$ and $\Pi : 3x + 2y - 5z + 1 = 0,$

(h) $L : \begin{cases} x = 2t \\ y = 1 - t \\ z = 3 + t \end{cases}, t \in \mathbf{R},$ and $\Pi : x + y - z - 2 = 0,$

(i) $L : x = y = z,$ and $\Pi : x + y + z - 2 = 0.$

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