Logarithms

1. Express as a single logarithm
   a) $4\log_7 5 - \frac{1}{6}\log_7 2 + 2\log_7 3$,
   b) $3 - 4\log x + \frac{1}{2}\log y$,
   c) $-\ln 3 - \frac{3}{4}\ln 5 - 2$,
   d) $\log_4 5 + 3\log_2 3 - \log_{\sqrt{2}} 9$,
   e) $\log_3 \sqrt[7]{9}\, 2 + \log_{729} 5 - 2\log_{\frac{1}{27}} 4 - 1$.

2. If $x = \log_3 a$, $y = \log_3 b$ and $z = \log_3 c$ express, in terms of x, y, z,
   a) $\log_3(ab^4\sqrt{c})$,
   b) $\log_9 \dfrac{\sqrt[3]{a}\,\sqrt[5]{b^4}}{c^{11}}$,
   c) $\log_{\frac{1}{81}}(ab^2 c^{-2})$.

3. If $x \in \mathbf{R}$ and $n \in \mathbf{N}$ solve, exactly, the equations below.
   a) $4^x = 5$.
   b) $e^x = 3$.
   c) $\left(\frac{2}{3}\right)^x = 5$.
   d) $5^x = 0$.
   e) $11^x = -2$.
   f) $(-2)^n = -32768$.
   g) $(-3)^n = 6561$.
   h) $\left(-\dfrac{2}{3}\right)^n = -\dfrac{2048}{177147}$.
   i) $(-5)^n = 78125$.
   j) $(-4)^n = -65536$.
   k) $\log_2 x = 3$.
   l) $\log x = -0.3$.
   m) $\ln x = 3$.
   o) $\log_{\frac{1}{2}} x = 5$.
   p) $\log_x 2 = 3$.
   q) $\log_x 3 = 2$.

4. Solve, exactly, the inequalities below.
   a) $2^x > 6$.
   b) $0.3^x \geq 2$.
   c) $(\ln 2)^x < 1$.
   d) $174^x \leq 0$.
   e) $0.4^x \geq -2$.
   f) $\pi^x < -0.0001$.
   g) $3^x > -1$.
   h) $\log x \geq -3$.
   i) $\log_6 x \leq \dfrac{16}{5}$.
   j) $\log_{0.8} x > 2$.
   k) $\ln x < -4$.