Polynomials

1. Factorize the polynomials below.

   a) $x^3 - 19x + 30$

   b) $2x^3 + 3x^2 + 2x + 1$

   c) $-\frac{1}{2}x^3 + x - \frac{1}{2}$

   d) $5x^3 - 24x^2 + 36x - 16$

   e) $\frac{1}{9}x^3 - x^2 + 3x - 3$

   f) $x^4 - 3x^3 - 4x^2 + 18x - 12$

   g) $x^4 + x^3 - x - 1$

   h) $-9x^4 - 3x^3 + 23x^2 - 13x + 2$

   i) $8x^4 + 20x^3 - 42x^2 + 23x - 4$

   j) $16x^4 + 32x^3 + 24x^2 + 8x + 1$

   k) $x^5 - 2x^4 - 4x^3 + 4x^2 - 5x + 6$

   l) $x^5 - x^4 - 2x^3 + 5x^2 - 5x + 2$

   m) $x^6 + 5x^5 + 7x^4 - 2x^3 - 13x^2 - 11x - 3$

2. The following polynomials have integer repeated roots. Find their multiplicity.

   a) $x^5 - 4x^4 + 3x^3 + 5x^2 - 8x + 4$

   b) $x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

   c) $x^6 - 4x^5 + 7x^4 - 8x^3 + 7x^2 - 4x + 1$

   d) $x^7 + 3x^6 + 3x^5 + 2x^4 + 5x^3 + 9x^2 + 7x + 2$

3. Factorize the following quartic polynomials over the real number field.

   a) $x^4 - 3x^2 + 2$

   b) $x^4 + x^2 - 2$

   c) $2x^4 + 3x^2 + 1$

   d) $x^4 - 6x^2 + 9$

   e) $\frac{1}{2}x^4 + x^2 + \frac{1}{2}$

   f) $(\star)$ $x^4 + 9$

   g) $(\star)$ $x^4 - x^2 + 25$

   h) $(\star)$ $3x^4 + 5x^2 + 9$