Trigonometric equations and inequalities

1. Solve, exactly, the equations below.
   (a) $\sin x = \frac{\sqrt{3}}{2}$ in $[0, 2\pi]$.
   (b) $\cos x = -\frac{1}{2}$ in $[-\pi, 2\pi]$.
   (c) $\tan x = 1$ in $[0, 3\pi]$.
   (d) $\sin x = 1$ in $[-120°, 240°]$.
   (e) $\cos x = \frac{\sqrt{2}}{2}$ in $[-300°, 30°]$.
   (f) $\tan x = -\sqrt{3}$ in $[-180°, 360°]$.
   (g) $\sin x = 0.4$ in $[0, 2\pi]$.
   (h) $\cos x = \frac{1}{4}$ in $[-\pi, 3\pi]$.
   (i) $\tan x = 7$ in $[0, 2\pi]$.
   (j) $\sin x = -\ln 2$ in $[-180°, 180°]$.
   (k) $\cos x = -\frac{\sqrt{2}}{6}$ in $[0°, 500°]$.
   (l) $\tan x = \frac{1}{2}$ in $[-360°, 120°]$.
   (m) $\sin(2x - \frac{\pi}{3}) = \frac{\sqrt{2}}{2}$ in $[0, 2\pi]$.
   (n) $\cos(\frac{1}{2}x - \frac{\pi}{4}) = -1$ in $[-2\pi, 3\pi]$.
   (o) $\tan(\frac{\pi}{5} - x) = \frac{\sqrt{3}}{3}$ in $[0, 2\pi]$.
   (p) $\sin(\frac{2}{3}x) = -\frac{1}{2}$ in $[-180°, 180°]$.
   (q) $\cos(x + 80°) = \frac{\sqrt{2}}{2}$ in $[-360°, 360°]$.
   (r) $\tan(2x - 100°) = 2$ in $[0°, 180°]$.

2. Solve, exactly, the equations below.
   (a) $\sin x = \sin(2x + \frac{3\pi}{4})$ in $[0, 2\pi]$.
   (b) $\cos x = \cos(x + \frac{\pi}{5})$ in $[-2\pi, 2\pi]$.
   (c) $\tan(2x) = \tan(\frac{5\pi}{6} - 3x)$ in $[0, \pi]$.
   (d) $\sin(x - 140°) = \sin(x + 220°)$ in $[-1000°, 1000°]$.
   (e) $\cos(x + 80°) = \cos(x - 80°)$ in $[-360°, 360°]$.
   (f) $\tan(x + 80°) = \tan(x - 80°)$ in $[0°, 180°]$.

3. Solve, exactly, the equations below.
   (a) $4\sin x = 5\cos x$ in $[0, 2\pi]$.
   (b) $\tan^2 x = 3$ in $[-180°, 180°]$.
   (c) $2\sin^2 x - 3\sin x + 1 = 0$ in $[0°, 360°]$.
   (d) $\sqrt{2}\cos^2 x + \cos x = 0$ in $[-\pi, \pi]$.
   (e) $\sin(2x) = 2\cos x$ in $[0, \pi]$.
   (f) $\cos(2x) = 3 - 2\cos x$ in $[-100°, 200°]$.
   (g) $\sin x = 4\tan x$ in $[-2\pi, 2\pi]$.
   (h) $\cos x = 1.5\tan x$ in $[0°, 180°]$.
   (i) $\tan(2x) = 2\tan x$ in $[0°, 180°]$.
   (j) $\sin(3x) = 2\sin x$ in $[0, 2\pi]$.

4. Solve, exactly, the equations below.

    (a) $\sin x + \cos x = 1$ in $[-\pi, \pi]$.

    (b) $\sin x - \cos x = 1$ in $[0°, 360°]$.

    (c) $\sin x + \sqrt{3}\cos x = 1$ in $[-\pi, \pi]$.

    (d) $\sin x - \sqrt{3}\cos x = 1$ in $[-180°, 180°]$.

    (e) $\sqrt{3}\sin x + \cos x = 1$ in $[-\pi, \pi]$.

    (f) $\sqrt{3}\sin x - \cos x = 1$ in $[0°, 360°]$.

    (g) $3\sin x + 4\cos x = 5$ in $[0°, 360°]$.

    (h) $3\sin x - 4\cos x = 1$ in $[-\pi, \pi]$.

5. Solve, exactly, the inequalities below.

    (a) $\sin x < \frac{\sqrt{3}}{2}$ in $[0, 2\pi]$.

    (b) $\cos x \geq \frac{1}{2}$ in $[-\pi, \pi]$.

    (c) $1 \leq \tan x < \sqrt{3}$ in $[0, 3\pi]$.

    (d) $\sin(\frac{3}{4}x) > -\frac{1}{2}$ in $[-180°, 180°]$.

    (e) $\sin x + \sqrt{3}\cos x \leq 1$ in $[-\pi, \pi]$.

    (f) $2\sin^2 x + 3\sin x + 1 > 0$ in $[0°, 360°]$.

    (g) $\cos^2 x - \cos x \leq 0$ in $[0, 4\pi]$.

    (h) $2\sin x < \tan x$ in $[0°, 360°]$.

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