Linear Differential Game With Two Pursuers and One Evader

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Abstract

The purpose of this paper is to compute two on one differential game No Escape Zones (NEZ) [8]. The main objective (over the scope of this paper) consists in designing suboptimal strategies in many on many engagements. 1x1 NEZ and 2x1 NEZ are components involved in suboptimal approaches we propose (i.e. “Moving Horizon Hierarchical Decomposition Algorithm” [6]). Several specific two pursuers one evader differential games have been already studied ([4], [7]). Nevertheless, we propose to compute 2x1 NEZ from 1x1 DGL/1 NEZ because DGL models are games with well defined analytical solutions [12].

We consider head on scenarios with two pursuers (P1 and P2) and one evader (E). First we summarize results about one on one pursuit evasion game using DGL/1 models. DGL/1 differential games are coplanar interceptions with constant velocities, bounded controls assuming small motion variations around the collision course triangle. Under this assumption the kinematics are linear. Each player is represented as a first order system (time lag constant). The criterion is the terminal miss distance (terminal cost only, absolute value of the terminal miss perpendicular to the initial Line Of Sight, LOS). DGL/1 are fixed time duration differential games, with final time defined by the closing velocity (assumed constant) and the initial pursuer evader range. The terminal projection procedure [3] allows to reduce the initial four dimension state vector representation to a scalar representation and to represent the optimal trajectories in the ZEM (Zero Effort Miss), Tgo (Time to Go) coordinate frame. The (ZEM, Tgo) frame is divided into two regions, the regular area and the singular one. For some appropriated differential game parameters (pursuer to evader maximum acceleration ratio μ and evader to pursuer time lag ratio ε), the singular area plays the role of capture zone so called also NEZ (leading to zero terminal miss), whilst the regular area corresponds to the non capture zone. The NEZ can be bounded (closed) or unbounded (open). The natural optimal strategies are bang-bang controls corresponding to the sign of ZEM (some refinements exist when defining optimal controls inside the NEZ). We start the 2x1 DGL/1 analysis with unbounded 1x1 NEZ as pictured in Figure 1 (NEZ delimited by the two plain red lines, non capture zone corresponding to the state space filled with optimal trajectories in dot blue lines). Moreover, we first assume same Tgo in each DGL/1 game (same initial range, same velocity for each pursuer).

Figure 1 : Unbounded NEZ (ZEM, Tgo frame)

When considering two pursuers and one evader (in the framework of DGL representations), some cases are easy to solve. If E is “above” (or “below”) the two pursuers (in ZEM, Tgo frame), the optimal evasion respect to each pursuer (considered alone) and according to both pursuers together is to turn right (to turn left) with maximum acceleration. In these cases (E “above” or “below”), there are

no changes in evasion trajectory (optimal strategies) by adding a pursuer. Therefore, the value of the 2x1 game is just the minimum of the two terminal miss distances.

$$J_{2x1}(u_1, u_2, v) = \min \{J(u_1, v), J(u_2, v)\}$$

With J the terminal miss, u_i the control of P_i , $i = \{1, 2\}$ and v the evader control.

$J(u_1, v)$: P1 E DGL/1 terminal miss

$J(u_2, v)$: P2 E DGL/1 terminal miss

$J_{2x1}(u_1, u_2, v)$: 2x1 DGL/1 outcome

For the other initial conditions (see Figure 2) we have to refine the optimal evasion behaviour. If E is between P1 and P2 (as described in Figure 2) then the optimal 2x1 evasion is a trade off between the incompatible P1 E and P2 E optimal escapes. Notice that in Figure 2 the initial LOS (in each DGL/1 game) have been chosen parallel.

Figure 2 : 2x1 game with Evader between the two Pursuers (ZEM, Tgo frame)

We compute the 2x1 DGL1 game solution ($J_{2x1}(u_1^*, u_2^*, v_{2x1}^*)$) considering that the optimal evasion control is a constant value during the entire game and considering that the terminal miss distances respect to P1 and P2 have to be the same. In the case of Figure 2, $v_1^* = -A_{E \max}$ (left turn), $v_2^* = A_{E \max}$ (right turn, $\pm A_{E \max}$ being the evader control bounds) and $-A_{E \max} \leq v_{2x1}^* \leq A_{E \max}$.

$$\begin{aligned} & J(u_1^*, v_2^*) \\ & J(u_2^*, v_1^*) \leq J_{2x1}(u_1^*, u_2^*, v_{2x1}^*) = J(u_1^*, v_{2x1}^*) = J(u_2^*, v_{2x1}^*) \leq \begin{matrix} J(u_1^*, v_1^*) \\ J(u_2^*, v_2^*) \end{matrix} \\ & J_{2x1}(u_1^*, u_2^*, v) \end{aligned}$$

This paper proposes

- to analyse the 2x1 game solution when the 1x1 DGL/1 NEZ are bounded, unbounded (different μ , ε parameters);
- to compute the 2x1 game solution when the 1x1 DGL/1 Tgo are equal and when they are not;
- to check the linearization assumptions by running the 2x1 solution in 2D non linear simulations;
- and to compare the 2x1 DGL approach to a 2x1 solution obtained using Linear Quadratic (LQ) approaches [11].

When the Tgo are different in the two DGL/1 games, we notice that as soon as a $P_i E$ game ends, the evader switches from the trade off control to the optimal strategy corresponding to the unique remaining pursuer (maximum control). For the LQ approach, we have to solve time varying Riccati

equations including impulse functions to take into account the case of different Tgo. Analytical Riccati solutions are easy to calculate. However, some sufficient conditions for existence of a solution have to be verified. The optimality condition for two person games is the existence of saddle point. For N person games, the optimality notion is the “non inferior Nash equilibrium”, which is identical to saddle point condition for two person games [1].

To conclude we ask the problem of transposing these results to 3D engagements. Moreover, we propose to compare this way of doing to other approaches (algorithms) which address the problem of computing, approaching or over approximating reachable sets with non linear kinematics (level set methods [10], victory domains from viability theory [5]). In a general manner, we would like to consider other approaches dedicated to suboptimal multi player strategies : reflection of forward reachable sets [9], minimization / maximization of the growth of particular level set functions ([13], [14]), LQ approach with evader terminal constraints and specific guidance law (Proportional Navigation) for the pursuers [2].

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