Riemann-Liouville, Caputo, and Sequential Fractional Derivatives in Differential Games

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Abstract

There exists a number of definitions of the fractional order derivative. The classical one is the definition by Riemann-Liouville [1]. The Riemann-Liouville fractional derivatives have a singularity at zero. That is why differential equations involving these derivatives require initial conditions of special form lacking clear physical interpretation. These shortcomings do not occur with the regularized fractional derivative in the sense of Caputo. Both the Riemann-Liouville and Caputo derivatives possess neither semigroup nor commutative property. That is why so-called sequential derivatives were introduced by Miller and Ross [2].

In this paper we treat sequential derivatives of special form [3]. Their relation to the Riemann-Liouville and Caputo fractional derivatives and to each other is established.

Differential games for the systems with the fractional derivatives of Riemann-Liouville, Caputo, as well as with the sequential derivatives are studied. Representations of such systems' solutions involving the Mittag-Leffler generalized matrix functions [4] are given. The use of asymptotic representations of these functions in the framework of the Method of Resolving Functions [5, 6, 7] allows to derive sufficient conditions for solvability of corresponding game problems. These conditions are based on the modified Pontrjagin's condition [5]. The results are illustrated on a model example where a dynamic system of order π pursues another system of order e .

References

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