1. $I(a)=\int_0^\infty e^{-ax}\sin^2\frac{x}{2}\,dx = \frac12\int_0^\infty e^{-ax}(1-\cos x)\,dx = \frac12\,\frac{e^{-ax}}{-a}\Big|_0^\infty - \frac12\int_0^\infty e^{-ax}\cos x\,dx \mp$

$$\left\{\begin{aligned} &\int_0^\infty e^{-ax}\cos x\,dx = \int_0^\infty e^{-ax}(\sin x)'\,dx = e^{-ax}\sin x\Big|_0^\infty + a\int_0^\infty e^{-ax}\sin x\,dx = \\ &\quad = a\left(e^{-ax}(-\cos x)\Big|_0^\infty + a\int_0^\infty e^{-ax}(-\cos x)\,dx\right) = a - a^2\int_0^\infty e^{-ax}\cos x\,dx \\ &\Rightarrow \int_0^\infty e^{-ax}\cos x\,dx = \frac{a}{a^2+1} \end{aligned}\right.$$

$$I(a) = \frac{1}{2a} - \frac12\,\frac{a}{a^2+1} = \frac{1}{2(a^3+a)}$$

$$J = \int_0^\infty \frac{e^{-\frac{x}{2}}-e^{-2x}}{x}\sin^2\frac{x}{2}\,dx = \int_0^\infty \int_{\frac12}^2 e^{-ax}\,da\,\sin^2\frac{x}{2}\,dx \overset{\dots}{=} \int_{\frac12}^2\int_0^\infty e^{-ax}\sin^2\frac{x}{2}\,dx\,da =$$

$$= \int_{\frac12}^2\left(\frac{1}{2a} - \frac12\,\frac{a}{a^2+1}\right)da = \left(\frac12\ln|a| - \frac14\ln(a^2+1)\right)\Big|_{\frac12}^2 = \frac12\ln 2 - \frac14\ln 5 + \frac12\ln 2 + \frac14\ln\frac54 =$$

$$= \ln 2 - \frac14\ln 4 = \frac12\ln 2 = \ln\sqrt2$$

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2. $\gamma(t) = \left(t, \sqrt t - \frac{t^{3/2}}{3}\right)$, $t\in[0,3]$

$\gamma'(t) = \left(1, \frac{1}{2\sqrt t} - \frac12\sqrt t\right)$ $\quad \|\gamma'(t)\| = \sqrt{1+\left(\frac{1}{2\sqrt t}-\frac12\sqrt t\right)^2} = \sqrt{\left(\frac{1}{2\sqrt t}+\frac12\sqrt t\right)^2} = \frac{1}{2\sqrt t}+\frac{\sqrt t}{2}$,

$M = \int_\gamma 1\,dl = \int_0^3\|\gamma'(t)\|\,dt = \int_0^3\left(\frac{1}{2\sqrt t}+\frac{\sqrt t}{2}\right)dt = \left(\sqrt t + \frac13 t^{3/2}\right)_0^3 = \sqrt3+\sqrt3 = 2\sqrt3$

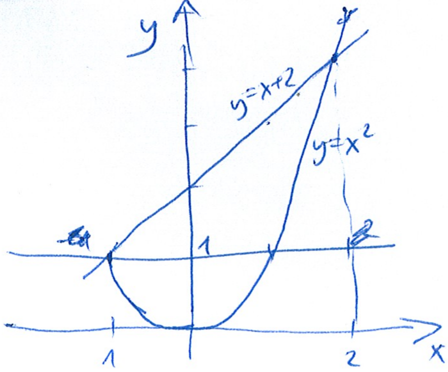
$M_x = \int_\gamma y\,dl = \int_0^3 \left(\sqrt t - \frac{t^{3/2}}{3}\right)\left(\frac{1}{2\sqrt t}+\frac{\sqrt t}{2}\right)dt = \int_0^3\left(-\frac{t^2}{6}+\frac t3+\frac12\right)dt = \left(-\frac{t^3}{18}+\frac{t^2}{6}+\frac12 t\right)_0^3 =$

$= -\frac32+\frac32+\frac32=\frac32$

$M_y = \int_\gamma x\,dl = \int_0^3 t\left(\frac{1}{2\sqrt t}+\frac{\sqrt t}{2}\right)dt = \left(\frac13 t^{3/2}+\frac15 t^{5/2}\right)\Big|_0^3 = \sqrt3+\frac95\sqrt3 = \frac{14}{5}\sqrt3$

$S = \left(\frac{M_y}{M}, \frac{M_x}{M}\right) = \left(\frac75, \frac{\sqrt3}{4}\right).$

3.
$$\int_{-1}^{2}\int_{x^2}^{x+2}\sqrt{y}\,dy\,dx = \left(\int_0^1\int_{-\sqrt{y}}^{\sqrt{y}} + \int_1^4\int_{y-2}^{\sqrt{y}}\right)\sqrt{y}\,dx\,dy$$



$$L=\int_{-1}^{2}\left(\frac{2}{3}y^{3/2}\right)\Big|_{x^2}^{x+2}dx=\int_{-1}^{2}\left(\frac{2}{3}(x+2)^{3/2}-\frac{2}{3}|x|^3\right)dx=$$

$$=\frac{2}{3}\cdot\frac{2}{5}(x+2)^{5/2}\Big|_{-1}^{2}-\frac{2}{3}\left(2\cdot\int_0^1 x^3dx+\int_1^2 x^3dx\right)=$$

$$=\frac{4}{15}\left(4^{5/2}-1\right)-\frac{2}{3}\left(2\cdot\frac{1}{4}+\frac{1}{4}(16-1)\right)=\frac{4}{15}(31)-\frac{2}{3}\left(\frac{1}{2}+\frac{15}{4}\right)=\frac{124}{15}-\frac{17}{6}=\frac{248-85}{30}=\frac{163}{30}$$

$$P=\int_0^1 2\sqrt{y}\cdot\sqrt{y}\,dy+\int_1^4\sqrt{y}\left(\sqrt{y}-y+2\right)dy=1+\left(\frac{y^2}{2}-\frac{2}{5}y^{5/2}+\frac{4}{3}y^{3/2}\right)\Big|_1^4=$$

$$=1+\left(\frac{15}{2}-\frac{2}{5}\cdot 31+\frac{4}{3}\cdot 7\right)=\frac{30+225-372+70\cdot 4}{30}=\frac{225-62}{30}=\frac{163}{30}.$$

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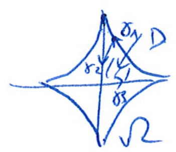
4. I $\quad D:\ x\geq 0,\ y\geq 0,\ x^{2/3}+y^{2/3}\leq 1 \iff D:\ \varphi\in[0,\frac{\pi}{2}],\ r\in[0,1]$

$x=r\cos^3\varphi,\ y=r\sin^3\varphi$

$$dx\,dy=\left|\det\begin{bmatrix}\cos^3\varphi & -3\cos^2\varphi\sin\varphi\cdot r\\ \sin^3\varphi & 3\sin^2\varphi\cos\varphi\cdot r\end{bmatrix}\right|dr\,d\varphi=\left|3r\left(\cos^4\varphi\sin^2\varphi+\cos^2\varphi\sin^4\varphi\right)\right|dr\,d\varphi=3r\cos^2\varphi\sin^2\varphi\,dr\,d\varphi$$

$$|D|=\int_0^1\int_0^{\pi/2}3r\cos^2\varphi\sin^2\varphi\,d\varphi\,dr=\frac{3r^2}{2}\Big|_0^1\cdot\frac{1}{4}\int_0^{\pi/2}\sin^2 2\varphi\,d\varphi=\frac{3}{2}\cdot\frac{1}{4}\cdot\frac{\pi}{4}=\frac{3\pi}{32}.$$



II
$$\iint_D 1\,dx\,dy=\frac{1}{4}\iint_\Omega\frac{1}{2}\left(\frac{\partial}{\partial x}x-\frac{\partial}{\partial y}(-y)\right)dx\,dy=\frac{1}{8}\int_\gamma(-y)\,dx+x\,dy=$$

$$=\frac{1}{8}\int_0^{2\pi}\left((-\sin^3t)\,3\cos^2t\cdot(-\sin t)+\cos^3t\cdot 3\sin^2t\cos t\right)dt=$$

$$=\frac{1}{8}\cdot 3\int_0^{2\pi}\sin^2t\cos^2t\,dt=\frac{3}{32}\int_0^{2\pi}\sin^2 2t\,dt=\frac{3\pi}{32}.$$

$\gamma(t)=(\cos^3t,\sin^3t)$, $t\in[0,2\pi)$

$\gamma_1(t)=\gamma(t)$, $t\in[0,\frac{\pi}{2}]$

$\gamma_2(t)=(0,1-t)$, $t\in[0,1]$

$\gamma_3(t)=(t,0)$, $t\in[0,1]$

Albo:
$$=\frac{1}{2}\left(\int_0^{\pi/2}\left(3\cos^2t\sin^4t+3\cos^4t\sin^2t\right)dt+\int_0^1 0\,dt+\int_0^1 0\,dt\right)=\frac{1}{2}\cdot 3\cdot\int_0^{\pi/2}\sin^2t\cos^2t\,dt=\frac{3\pi}{32}$$