Mathematical Analysis - List 10

1. Evaluate the indefinite integral.

a) $\int \left(3\sqrt[3]{x^2} + \frac{1}{x^3} - 2x\sqrt{x}\right)\, dx$; b) $\int \frac{(1-x)\, dx}{1-\sqrt[3]{x}}$; c) $\int \frac{x^4\, dx}{x^2+1}$;

d) $\int \frac{\cos 2x\, dx}{\cos x - \sin x}$; e) $\int \frac{x^3 + \sqrt[3]{x^2} - 1}{\sqrt{x}}\, dx$; f) $\int \frac{2^x - 5^x}{10^x}\, dx$.

2. Evaluate the indefinite integral by making an appropriate substitution.

a) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$; b) $\int \frac{3x}{\sqrt{x^2+1}}\, dx$; c) $\int (x+1)\sin\left(x^2+2x+2\right)\, dx$;

d) $\int \frac{\cos x\, dx}{\sqrt{1+\sin x}}$; e) $\int \frac{(3x+2)\, dx}{3x^2+4x+7}$; f) $\int \frac{e^x\, dx}{e^{2x}+1}$;

g) $\int \frac{5\sin x\, dx}{3-2\cos x}$; h) $\int x^3 e^{x^2}\, dx$; i) $\int \sin^3 x\, dx$.

3. Use integration by parts to evaluate the indefinite integral.

a) $\int \ln(x+1)\, dx$; b) $\int x^2 2^x\, dx$; c) $\int x^2 \sin x\, dx$;

d) $\int e^{2x}\sin x\, dx$; e) $\int x\ln x\, dx$; f) $\int \arccos x\, dx$.

4. Evaluate the indefinite integral.

a) $\int \frac{\sin x}{1+\cos^2 x}\, dx$; b) $\int \frac{1+x}{1+x^2}\, dx$; c) $\int x^3\sqrt{x^2+5}\, dx$;

d) $\int \sin^4 x\cos x\, dx$; e) $\int \sin^5 x\cos^2 x\, dx$; f) $\int \sin x\sin(\cos x)\, dx$.

5. Express the integrand as a sum of partial fractions and find the integral.

a) $\int \frac{(x+2)\, dx}{x(x-2)}$; b) $\int \frac{x^2\, dx}{x+1}$; c) $\int \frac{dx}{(x-1)x^2}$;

d) $\int \frac{dx}{(x^2+1)(x^2+4)}$; e) $\int \frac{(4x+1)\, dx}{2x^2+x+1}$; f) $\int \frac{(3x-1)\, dx}{x^2-x+1}$.

6. Evaluate the indefinite integral.

a) $\int (|x|+1)\, dx$; b) $\int \left|1-x^2\right|\, dx$;

c) $\int |\cos x|\, dx, \quad x \in [0,\pi]$.