Mathematical Analysis - List 11

1. Evaluate the following integrals by interpreting each in terms of areas.

a) $\int_{-2}^{2} (2+x)\,dx$; b) $\int_{-2}^{0} \sqrt{4-x^2}\,dx$; c) $\int_{0}^{3} (2x+\sqrt{9-x^2})\,dx$.

2. Set up an expression for $\int_{-1}^{3} f(x)dx$ as a limit of Riemann sums taking the sample points to be right-hand endpoints of subintervals. Next evaluate the limit.

a) $f(x) = 2-x$; b) $f(x) = (x+1)^2$; c) $f(x) = e^x$;

3. Express the following limit as a definite integral.

a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5}$; b) $\lim_{n\to\infty} \frac{\pi}{4n} \sum_{i=1}^{n} \operatorname{tg} \frac{\pi i}{4n}$;

c) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1+\frac{3i}{n}}$; d) $\lim_{n\to\infty} \left\{ \frac{1}{n} \ln \left[\frac{(1+n)(2+n)\cdot\ldots\cdot(n+n)}{n^n} \right] \right\}$.

4. Show that $\int_{1}^{4} \sqrt{x^2+1}\,dx \geqslant 3\sqrt{2}$. Next show that $\sqrt{1+x^2} \geqslant x$ and by means of this inequality get the following (better!) estimate $\int_{1}^{4} \sqrt{x^2+1}\,dx \geqslant 7.5$.

5. Find the interval $[a,b]$ for which the value of the integral $\int_{a}^{b} (2+x-x^2)\,dx$ is a maximum.

6. Find the derivative of the given function.

a) $G(x) = \int_{\sqrt{x}}^{x^2+1} t^2 \cos t\,dt$; b) $H(x) = \int_{\operatorname{tg} x}^{3} \sin(t^4)\,dt$.

7. Evaluate the integral.

a) $\int_{1}^{2} \left(\sqrt[3]{x} + \frac{1}{\sqrt[4]{x}} \right) dx$; b) $\int_{0}^{1} \frac{x-1}{x+1}\,dx$; c) $\int_{1}^{8} \frac{x-1}{\sqrt[3]{x^2}}\,dx$;

d) $\int_{0}^{9} \frac{dx}{x^2+9}$; e) $\int_{e^{-1}}^{e} \ln x\,dx$; f) $\int_{0}^{\pi} \sin^2 x \cos x\,dx$.

8. Evaluate the integral.

a) $\int_{1}^{4} \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx$; b) $\int_{-a}^{a} x\sqrt{x^2+a^2}\,dx$; c) $\int_{e}^{e^4} \frac{dx}{x \ln x}$;

d) $\int_{0}^{\pi/2} e^{\sin x} \cos x\,dx$; e) $\int_{0}^{\pi/3} \frac{\sin x}{\cos^2 x}\,dx$; f) $\int_{0}^{1/2} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx$.