Mathematical Analysis - List 13

1. Find the average value of $f(x) = 5x^2\sqrt{1+x^3}$ on the interval $[0, 2]$.

2. If $f_{\text{ave}}[a, b]$ denotes the average value of f on the interval $[a, b]$ and $a < c < b$, show that

$$f_{\text{ave}}[a, b] = \frac{c-a}{b-a} f_{\text{ave}}[a, c] + \frac{b-c}{b-a} f_{\text{ave}}[c, b].$$

3. The linear density in a rod 8 m long is $12/\sqrt{x+1}$ kg/m, where x is measured in meters from one end of the rod. Find the average density of the rod.

4. Find the exact length of the curve.

a) $y = \sqrt{1-x^2}, \quad 0 \leqslant x \leqslant 1;$

b) $y = \ln(\cos x)$, $0 \leqslant x \leqslant \pi/4$.

5. Find the surface area of the solid of revolution obtained by rotating the curve about the specified axis.

a) $y = \sqrt{x+4}, \quad -4 \leqslant x \leqslant 2, \quad x$-axis; b) $y = \cos x, \quad 0 \leqslant x \leqslant \frac{\pi}{2}, \quad x$-axis;

c) $y = \ln x, \quad 1 \leqslant x \leqslant \sqrt{3}\,, \quad y$-axis.

6. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

a) $\displaystyle\int_{-\infty}^{0} e^{3x} dx;$ b) $\displaystyle\int_{0}^{\infty} xe^{-2x} dx;$ c) $\displaystyle\int_{-\infty}^{\infty} xe^{-3x^2} dx;$

d) $\displaystyle\int_{2}^{\infty} \frac{1}{(x+3)^{3/2}} dx;$ e) $\displaystyle\int_{2}^{\infty} \frac{1}{\sqrt{x+3}} dx;$ f) $\displaystyle\int_{-\infty}^{\infty} \frac{x}{x^2+1} dx;$

g) $\displaystyle\int_{e}^{\infty} \frac{1}{x(\ln x)^2} dx;$ h) $\displaystyle\int_{1}^{\infty} \frac{\ln x}{x^3} dx;$ i) $\displaystyle\int_{0}^{\infty} \frac{1}{\sqrt{x}(1+x)} dx.$

7. Use the Comparison Theorem to determine whether the integral is convergent or divergent

a) $\displaystyle\int_{1}^{\infty} \frac{\cos^2 x}{1+x^2} dx;$ b) $\displaystyle\int_{1}^{\infty} \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} dx;$ c) $\displaystyle\int_{1}^{\infty} \frac{1}{x(\sqrt{x}+1)} dx;$

d) $\displaystyle\int_{\pi}^{\infty} \frac{x+\sin x}{x^3} dx;$ e) $\displaystyle\int_{1}^{\infty} \frac{1}{x+e^{2x}} dx;$ f) $\displaystyle\int_{1}^{\infty} \frac{1}{\sqrt{x^3+1}} dx.$

8. Use the Limit Comparison Theorem to determine whether the integral is convergent or divergent

a) $\displaystyle\int_{1}^{\infty} \sin^2 \frac{1}{x} dx;$ b) $\displaystyle\int_{1}^{\infty} \frac{\sqrt{x}+1}{x(x+1)} dx.$