Mathematical Analysis - List 14

1. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

a) $\int_0^1 \frac{\ln x}{\sqrt{x}} dx$; b) $\int_0^3 \frac{1}{x\sqrt{x}} dx$.

2. Use the Comparison Theorem to determine whether the integral is convergent or divergent

a) $\int_0^{\pi/2} \frac{dx}{x \sin x}$; b) $\int_0^1 \frac{e^{-x}}{\sqrt{x}} dx$.

3. Show that $\int_0^\infty x^2 e^{-x^2} dx = \frac{1}{2} \int_0^\infty e^{-x^2} dx$.

4. Show that $\int_0^\infty e^{-x^2} dx = \int_0^1 \sqrt{-\ln y}\, dy$ by interpreting the integrals as areas.

5. Given the surface $(x-1)^2 + (y+3)^2 + (z-2)^2 = 4$.

a) Find the equations of the circles (if any) where the sphere intersects each coordinate plane.
b) Find the points (if any) where the sphere intersects each coordinate axis.

6. The temperature T at any point in the region $-10 \leqslant x \leqslant 10$, $-10 \leqslant y \leqslant 10$ is given by the function

$$T(x, y) = 100 - x^2 - y^2.$$

a) Sketch isothermal curves (curves of constant temperature) for $T = 100°\text{C}$, $T = 75°\text{C}$, $T = 50°\text{C}$, $T = 100°\text{C}$, $T = 25°\text{C}$, and $T = 0°\text{C}$.
b) Suppose a heat-seeking bug is put down at any point on the xy-plane. In which direction should it move to increase its temperature fastest? How is that direction related to the level curves through that points?

7. Sketch the graph of the function:

a) $f(x, y) = \sqrt{4 - (x-1)^2 - y^2}$; b) $f(x, y) = 4 - \sqrt{(x-1)^2 + y^2}$;

c) $f(x, y) = 4 + (x-1)^2 + (y+1)^2$; d) $f(x, y) = 4 - |x-1|$;

e) $f(x, y) = 4 + (x+1)^2$; g) $f(x, y) = 2\cos y$;

8. Find the domain of the function:

a) $f(x, y) = \frac{xy^2}{\sqrt{x^2 + y^2 - 16}}$; b) $f(x, y) = \ln \frac{x^2 + y^2 - 4}{9 - x^2 - y^2}$;

c) $f(x, y) = \frac{x - y}{(x-1)^2 + (y+1)^2}$; d) $f(x, y) = 4 - |x-1|$;