Mathematical Analysis - List 15

1. Let $f(x,y)=\dfrac{x^2}{x^2+y}$, $x^2+y\neq 0$. Examine what happens when $(x,y)\to(0,0)$ along the curve $y=kx^2$ for different values of $k\neq -1$ and show that f does not have a limit at $(0,0)$.

2. Let $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2} & \text{for} \quad (x,y)\neq(0,0)\\ 0 & \text{for} \quad (x,y)=(0,0)\end{cases}$.

a) Show that $f(0,y)$ and $f(x,0)$ are each continuous functions of one variable.
b) Show that rays emanating from the origin are contained in contours of f.
c) Is f continuous at $(0,0)$?

3. Let $f(x,y)=\begin{cases}\dfrac{xy(x^2-y^2)}{x^2+y^2} & \text{for} \quad (x,y)\neq(0,0)\\ 0 & \text{for} \quad (x,y)=(0,0)\end{cases}$. Compute $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.

4. Explain why the following function is not continuous along the line $y=0$. (At any point of the line?)

$$f(x,y)=\begin{cases}1-x, & y\geqslant 0,\\ -2, & y<0.\end{cases}$$

5. Determine whether there is a value for c making the function continuous everywhere. If so, find it. If not, explain why not.

a) $$f(x,y)=\begin{cases}c+y, & x\leqslant 3,\\ 5-y, & x>3,\end{cases}$$

b) $$f(x,y)=\begin{cases}c+y, & x\leqslant 3,\\ 5-x, & x>3.\end{cases}$$

6. Determine whether there is a value for a making the function continuous at $(0,0)$. If so, find it. If not, explain why not.

a) $$f(x,y)=\begin{cases}\dfrac{\sin xy}{y} & \text{when} \quad y\neq 0,\\ a & \text{when} \quad y=0,\end{cases}$$

b) $$f(x,y)=\begin{cases}\dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} & \text{for} \quad (x,y)\neq(0,0)\\ a & \text{for} \quad (x,y)=(0,0).\end{cases}$$