Mathematical Analysis - List 16

1. Find the indicated partial derivative. Assume the variables are restricted to a domain on which the function is defined.

a) $\dfrac{\partial F}{\partial m_2}$ and $\dfrac{\partial F}{\partial r}$ if $F = \dfrac{Gm_1m_2}{r^2}$; b) $\dfrac{\partial m}{\partial v}$ if $m = \dfrac{m_0}{\sqrt{1-v^2/c^2}}$;

c) z_x and z_y if $z = x^9 + 3^2y + x^y$; d) z_y if $z = \dfrac{4x^2y^7 - y^2}{15xy - 8y}$;

e) $\dfrac{\partial}{\partial \lambda}\left(\dfrac{x^2y\lambda - 3\lambda^5}{\sqrt{\lambda^2 - 3\lambda + 7}}\right)$; f) $\dfrac{\partial}{\partial w}\left(\dfrac{x^2yw - 3xy^3w^5}{3w^2+2}\right)^{-5/2}$.

2. Show that the function $Q(K, L) = bK^\alpha L^{1-\alpha}$, where $0 < \alpha < 1$, satisfies the equation

$$K\frac{\partial Q}{\partial K} + L\frac{\partial Q}{\partial L} = Q.$$

3. Compute all second-order partial derivatives.

a) $f(x,y) = \sin(x^2+y^2)$; b) $f(x,y) = xe^{xy}$; c) $f(x,y) = x + \dfrac{x}{y}$;

d) $f(x,y) = y\ln(xy)$; e) $f(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$; f) $f(x,y,z) = \ln(x^2+y^4+z^6+1)$.

4. Let $f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2} & \text{for} \quad (x,y) \neq (0,0) \\ 0 & \text{for} \quad (x,y) = (0,0) \end{cases}$. Compute $f_{xy}(0,0)$ and $f_{yx}(0,0)$

5. Compute

a) $\dfrac{\partial^3 f}{\partial y \partial x^2}$ for $f(x,y) = \cos(xy)$; b) $\dfrac{\partial^5 f}{\partial x \partial y^2 \partial z^2}$ for $f(x,y,z) = e^{xy+z}$.

6. Let $f(x,y) = \begin{cases} \dfrac{xy^2}{x^2+y^2} & \text{for} \quad (x,y) \neq (0,0) \\ 0 & \text{for} \quad (x,y) = (0,0) \end{cases}$.

a) Is f differentiable for $(x,y) \neq (0,0)$?
b) Find $f_x(0,0)$ and $f_y(0,0)$.
c) Is f differentiable at $(0,0)$?

7. Find the equation of the tangent plane to $z = \sqrt{17 - x^2 - y^2}$ at the point $(2,3,2)$.

8. Find the equation of the tangent plane to $z = \dfrac{8}{xy}$ at the point $(2,1,4)$.

9. At what point on the surface $z = 1 + x^2 + y^2$ is its tangent plane parallel to the following planes?
a) $z - 5 = 0$, b) $6x - 10y - z + 5 = 0$.

10. A differentiable function f has the property that $f(1,3) = 7$ and $\operatorname{grad} f(1,3) = 2\vec{\imath} - 5\vec{\jmath}$.
a) Find the equation of the tangent line to the level curve of f through the point $(1,3)$.
b) Find the equation of the tangent plane to the surface $z = f(x,y)$ at the point $(1,3,7)$.

11. Let $f(x,y) = (e^x - x)\cos y$. Find a vector which is perpendicular to the level curve of f through the point $(2,3)$ in the direction in which f decreases most rapidly.