Mathematical Analysis - List 17

1. Find the differential of the function at the given point.

a) $F(m,r) = \dfrac{Gm}{r^2}$ at $(100,10)$; b) $Q(K,L) = 1.02K^{0.75}L^{0.25}$ at $(100,1)$;

c) $f(x,y) = xe^{-2y}$ at $(2,0)$; d) $h(x,t) = e^{-3t}\sin(x+5t)$ at $(\pi/2,0)$.

2. Find the differential of $f(x,y) = \sqrt{x^2+y^3}$ at the point $(1,2)$. Use it to estimate $f(1.04,1.98)$.

3. Find the differential of $f(x,y) = x^y$ at the point $(2,3)$. Use it to estimate $f(2.04,2.97)$.

4. A pendulum of period T and length l was used to determine g from the formula

$$g = \frac{4\pi^2 l}{T^2} \qquad \text{and} \qquad l = s + \frac{k^2}{s}, \quad k < s.$$

If the measurements of k and s are accurate to within 1%, find the maximum percentage error in l. If the measurements of T is accurate to within 0.5%, find the maximum percentage error in the computed value of g.

5. What can you conclude about the behavior of f near the point (a,b) if f has continuous second-order partial derivatives at (a,b) and :

a) $f_x = f_y = 0$ and $f_{xx} > 0, f_{yy} > 0, f_{xy} = 0$ at (a,b);

b) $f_x = f_y = 0$ and $f_{xx} > 0, f_{yy} = 0, f_{xy} > 0$ at (a,b);

c) $f_x = f_y = 0$ and $f_{xx} > 0, f_{yy} < 0$ at (a,b).

6. Find the local maxima, local minima of the given function.

a) $f(x,y) = (x+y)(xy+1)$;

b) $f(x,y) = 1 - \cos x + y^2/2$;

c) $f(x,y) = x^3 + y^3 - 3xy$.

7. A missile has a remote guidance device which is sensitive to both temperature and humidity. If t is the temperature in °C and h is percent humidity, the range over which the missile can be controlled is given by:

$$R = 27800 - 5t^2 - 6ht - 3h^2 + 400t + 300h.$$

What are the optimal atmospheric conditions for controlling the missile?

8. An irrigation canal has an isosceles trapezoidal cross-section of area 50 m^2. The average flow rate in the canal is inversely proportional to the wetted perimeter, p, of the canal, that is, to the perimeter of the trapezoid, excluding the top. Thus to maximize the flow we must minimize p. Find the depth d, base width w, and the trapezoid's base angle α that give the maximum flow rate.

9. Find the minimum and maximum values of the function $f(x,y) = 4x^2 - xy + 4y^2$ over the closed disk $x^2 + y^2 \leqslant 2$.

10. Find the minimum and maximum values of the function $f(x,y) = \sin x + \sin y + \sin(x+y)$ over the closed square $0 \leqslant x \leqslant \frac{\pi}{2}$, $0 \leqslant y \leqslant \frac{\pi}{2}$