

Zad. 32

$$\begin{aligned} \hookrightarrow \frac{\partial^2 f}{\partial z \partial \bar{z}} &= 4 \cdot \left[\frac{1}{2} \left(\frac{\partial^2}{\partial x^2} - i \frac{\partial^2}{\partial y^2} \right) \right] \frac{\partial f}{\partial \bar{z}} = \\ &= 4 \cdot \left[\frac{1}{2} \cdot \frac{1}{2} \left(\frac{\partial^2 f}{\partial x^2} + i \frac{\partial^2 f}{\partial x \partial y} - i \left(\frac{\partial f}{\partial y \partial x} + i \frac{\partial^2 f}{\partial y^2} \right) \right) \right] = \\ &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + i \left(\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x} \right) = \\ &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \Delta f \end{aligned}$$