33. Teza: $\Delta f = 0$ na Ω, $f \in C^2(\Omega)$

$f = u + iv$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \Big/ \frac{\partial}{\partial x} \qquad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} \Big/ \frac{\partial}{\partial y}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial y \partial x} \qquad \frac{\partial^2 v}{\partial x \partial y} = -\frac{\partial^2 u}{\partial y^2}$$

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 v}{\partial y \partial x} - \frac{\partial^2 v}{\partial x \partial y} = 0$$

$$\Delta v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 u}{\partial y \partial x} = 0$$

1 Czyli f harmoniczna

II

$$\Delta f = 4 \frac{\partial}{\partial z} \underbrace{\frac{\partial f}{\partial \bar{z}}}_{=0} = 0 \rightarrow \Delta(\operatorname{Re} f) = \operatorname{Re}(\Delta f) = 0$$