

$f(z) = z \in H(\mathbb{C})$ , zatem  $\sum_{n=0}^{\infty} \frac{z^n}{n!} \in H(\mathbb{C})$ ,

podobnie  $\sin(z) = \frac{e^{iz} - e^{-iz}}{2i} \in H(\mathbb{C})$

i  $\cos(z) = \frac{e^{iz} + e^{-iz}}{2} \in H(\mathbb{C})$

$$\frac{\partial}{\partial z} e^z = \frac{\partial}{\partial z} \sum_{n=0}^{\infty} \frac{z^n}{n!} = \sum_{n=1}^{\infty} \frac{z^{n-1}}{(n-1)!} = \sum_{k=0}^{\infty} \frac{z^k}{k!} = e^z$$

$$\frac{\partial}{\partial z} \sin(z) = \frac{\partial}{\partial z} \frac{e^{iz} - e^{-iz}}{2i} = \frac{ie^{iz} + ie^{-iz}}{2i} = \frac{e^{iz} + e^{-iz}}{2} = \cos(z)$$

$$\frac{\partial}{\partial z} \cos(z) = \frac{\partial}{\partial z} \frac{e^{iz} + e^{-iz}}{2} = \frac{ie^{iz} - ie^{-iz}}{2} = \frac{i(e^{iz} - e^{-iz})}{2} = -\frac{e^{iz} - e^{-iz}}{2i} = -\sin(z)$$

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