$$\cos(z+w) = \frac{e^{i(z+w)} + e^{-i(z+w)}}{2} = \frac{2e^{i(z+w)} + 2e^{-i(z+w)}}{4} =$$

$$= \frac{e^{i(z+w)} - e^{i(z-w)} - e^{i(w-z)} + e^{-i(z+w)}}{4}$$

$$\left(e^{-i(z+w)} + \cancel{e^{i(z-w)}} + \cancel{e^{i(w-z)}} + e^{-i(z+w)}\right)\frac{1}{-4} =$$

$$= \underbrace{\frac{e^{iz} + e^{-iz}}{2} \cdot \frac{e^{+wi} + e^{-wi}}{2}} - \frac{e^{iz} - e^{-iz}}{2i} \cdot \underbrace{\frac{e^{iw} - e^{-iw}}{2i}} =$$

$$= \cos z \cdot \cos w - \sin z \cdot \sin w$$

$$\cos z \cdot \cos z + \sin z \cdot \sin z = \frac{(e^{iz} + e^{-iz})^2}{4} + \frac{(e^{iz} - e^{-iz})^2}{-4} =$$

$$= \frac{e^{2iz} + 2\frac{e^{iz}}{e^{iz}} + e^{-2iz} - \left(e^{2iz} - 2\frac{e^{iz}}{e^{iz}} + e^{-2iz}\right)}{4} = \frac{4}{4} = 1$$