$$e^z = e^{x+iy} = e^x(\cos y + i\sin y) = \underbrace{e^x\cos y}_{u} + i\underbrace{e^x\sin y}_{v}$$

1) $\frac{\partial u}{\partial x} = e^x\cdot\cos y = \frac{\partial v}{\partial y} = e^x\cdot\cos y$

2) $\frac{\partial u}{\partial y} = -e^x\cdot\sin y \qquad \frac{\partial v}{\partial x} = e^x\cdot\sin y$

$$\sin(x+iy) = \sin(x)\cos(iy) + \cos(x)\sin(iy) =$$

$$= \frac{e^{ix}-e^{-ix}}{2i}\cdot\frac{e^{-y}+e^{y}}{2} + \frac{e^{ix}+e^{-ix}}{2}\cdot\frac{e^{-y}-e^{y}}{2i} =$$

$$= \underbrace{\sin x\cdot\cosh y}_{u} + i\underbrace{\cos x\sinh y}_{v}$$

$$\frac{\partial u}{\partial x} = \cos x\cosh y \quad \not\Rightarrow \quad \frac{\partial v}{\partial x} = -\sin x\sinh y$$

$$\frac{\partial u}{\partial y} = \sin x\sinh y \qquad \frac{\partial v}{\partial y} = \cos x\cosh y$$

□