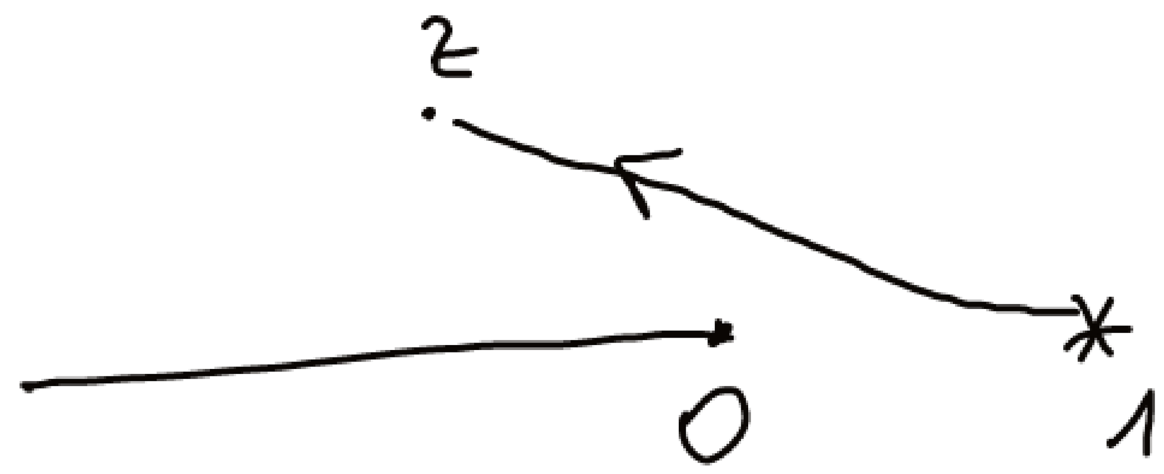
$$\mathrm{Log}\, z = \int_{\langle 1, z\rangle} \frac{dw}{w}$$



Tak jak w dowodzie tw. o funkcji pierwotnej można pokazać, że

$$(\mathrm{Log})'(z) = \frac{1}{z}$$

dla $f(w) = \frac{1}{w}$ i $\Omega = \mathbb{C} \setminus (-\infty, 0]$

$$\left(\frac{\exp(\mathrm{Log}\, z)}{z}\right)' = \frac{[\exp(\mathrm{Log}\, z)]' \cdot z - \exp(\mathrm{Log}\, z)}{z^2} =$$

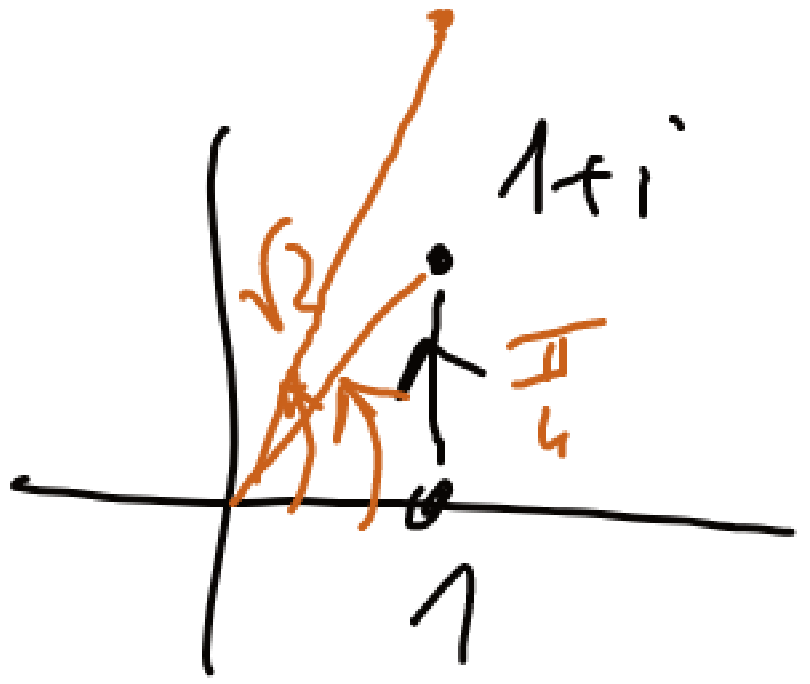
$$= \frac{e^{\mathrm{Log}\, z} \cdot \frac{1}{z} \cdot z - e^{\mathrm{Log}\, z}}{z^2} = 0$$

$$\mathrm{Log}\, 1 = \int_{\langle 1, 1\rangle} \frac{1}{w}\, dw = 0$$

$$\frac{\exp(\mathrm{Log}\, z)}{z} = \mathrm{const} = 1 \qquad \frac{\exp(\mathrm{Log}\, 1)}{1} = \exp(0) = 1$$

$$\Rightarrow \boxed{\exp(\mathrm{Log}\, z) = z}$$

$$\mathrm{Log}(1+i) = \int_{\langle 1, 1+i\rangle} \frac{dw}{w} = \int_0^1 \frac{i\,dt}{1+it} =$$



$\gamma(t) = 1+it,$

$t\in[0,1]$

$$= \int_0^1 \frac{i(1-it)}{1+t^2}\,dt = i\int_0^1 \frac{dt}{1+t^2} + \int_0^1 \frac{t}{1+t^2}\,dt =$$

$$= i\,\mathrm{arctg}\,t\Big|_0^1 + \frac{1}{2}\ln|1+t^2|\Big|_0^1 = i\cdot\frac{\pi}{4} + \frac{1}{2}\ln 2 = \underbrace{\frac{\pi}{4}i + \ln\sqrt{2}}$$