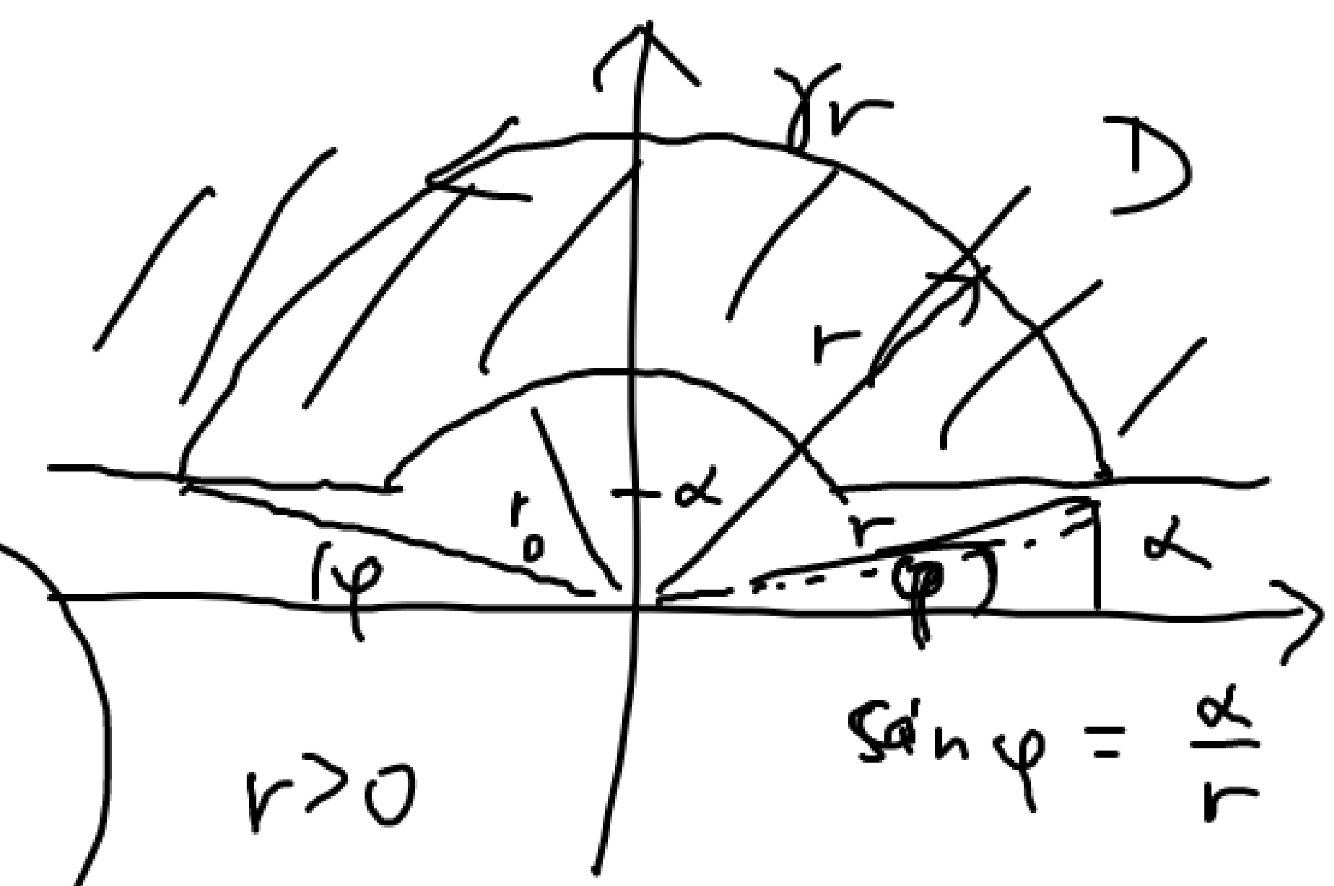
62 $\beta>0$, $\alpha\geq 0$, $D=\{z\in\mathbb{C} : |z|\geq r_0, \operatorname{Im} z\geq\alpha\}$, $f\in C(D)$, $f(z)\to 0$ dla $|z|\to\infty$

$$\Rightarrow \int_{\gamma_r} e^{i\beta z} f(z)\,dz \xrightarrow[r\to\infty]{} 0$$

$\left(|e^{\alpha+i\beta}|=e^{\alpha}\right.$ $(\alpha,\beta\in\mathbb{R})$



$r>0$

$\gamma_r(t)=re^{it}$, $t\in\left[\arcsin\frac{\alpha}{r}, \pi-\arcsin\frac{\alpha}{r}\right]=:[a(r),b(r)]$

$$\left|\int_{\gamma_r} e^{i\beta z} f(z)\,dz\right| = \left|\int_{a(r)}^{b(r)} e^{i\beta re^{it}} f(re^{it})\, r\underbrace{ie^{it}}\,dt\right| \leq$$

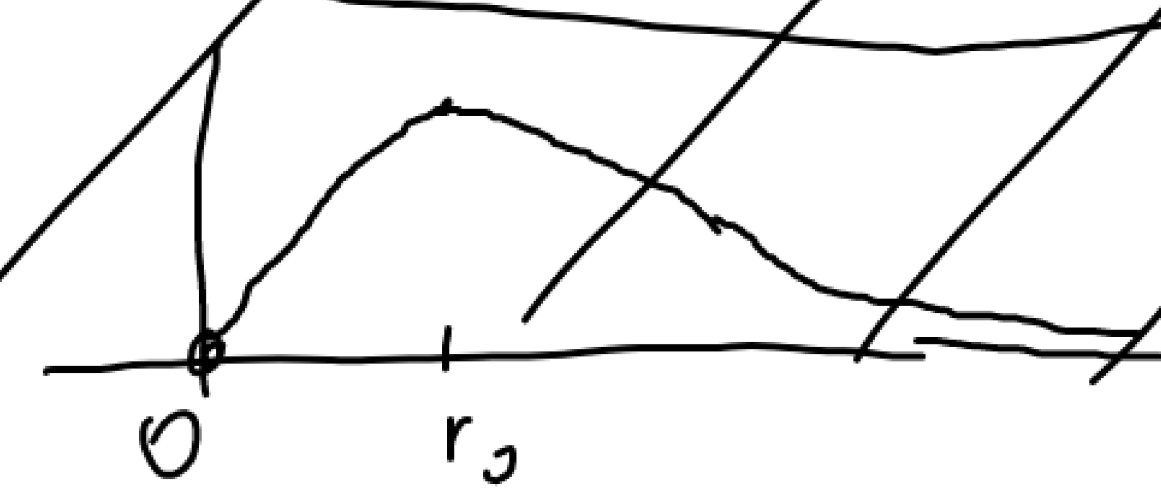
$$\leq \int_{a(r)}^{b(r)} \left|e^{i\beta r(\cos t+i\sin t)}\underbrace{f(re^{it})}\right|\cdot r\,dt \leq$$

$$\leq \sup_{\substack{x\in D\\ |x|=r}} |f(x)|\cdot\int_{a(r)}^{b(r)} e^{-\beta r\sin t}\, r\,dt \leq \underbrace{\sup_{\substack{x\in D\\ |x|=r}}|f(x)|}_{\downarrow 0}\cdot \underbrace{2\int_0^{\pi/2} e^{-\beta r\sin t}\, r\,dt}_{\leq M} \to 0$$

$g(r)=e^{-\beta r\sin t}\cdot r$ $\quad(t\in(0,\pi))$

$g'(r)=e^{-\beta r\sin t}\cdot(-\beta\sin t)\cdot r+e^{-\beta r\sin t}\cdot 1=e^{-\beta r\sin t}(1-r\beta\sin t)$

$r_0=\frac{1}{\beta\sin t} \Rightarrow g'(r_0)=0$



$0\leq g(r)\leq g\left(\frac{1}{\beta\sin t}\right)=e^{-1}\cdot\frac{1}{\beta\sin t}$

$$e^{-\beta r\sin t}\leq e^{-\beta r\cdot\frac{2}{\pi}t}$$

$\beta r\sin t\geq\frac{2}{\pi}t$, $t\in\left[0,\frac{\pi}{2}\right]$

$$\int_0^{\frac{\pi}{2}} e^{-\beta r\sin t}\, r\,dt\leq\int_0^{\pi/2} e^{-\beta r\frac{2}{\pi}t}\, r\,dt=$$

$$=\left|\begin{matrix} y=rt\\ dy=r\,dt\end{matrix}\right| = \int_0^{r\pi/2} e^{-\frac{\beta 2}{\pi}y}\,dy\leq\int_0^{\infty} e^{-y\frac{2\beta}{\pi}}\,dy=M<\infty$$