$\boxed{63}$

$$\int_\gamma \frac{f(z)}{(z-z_0)^{n+1}}\,dz = \frac{2\pi i\, f^{(n)}(z_0)}{n!}$$

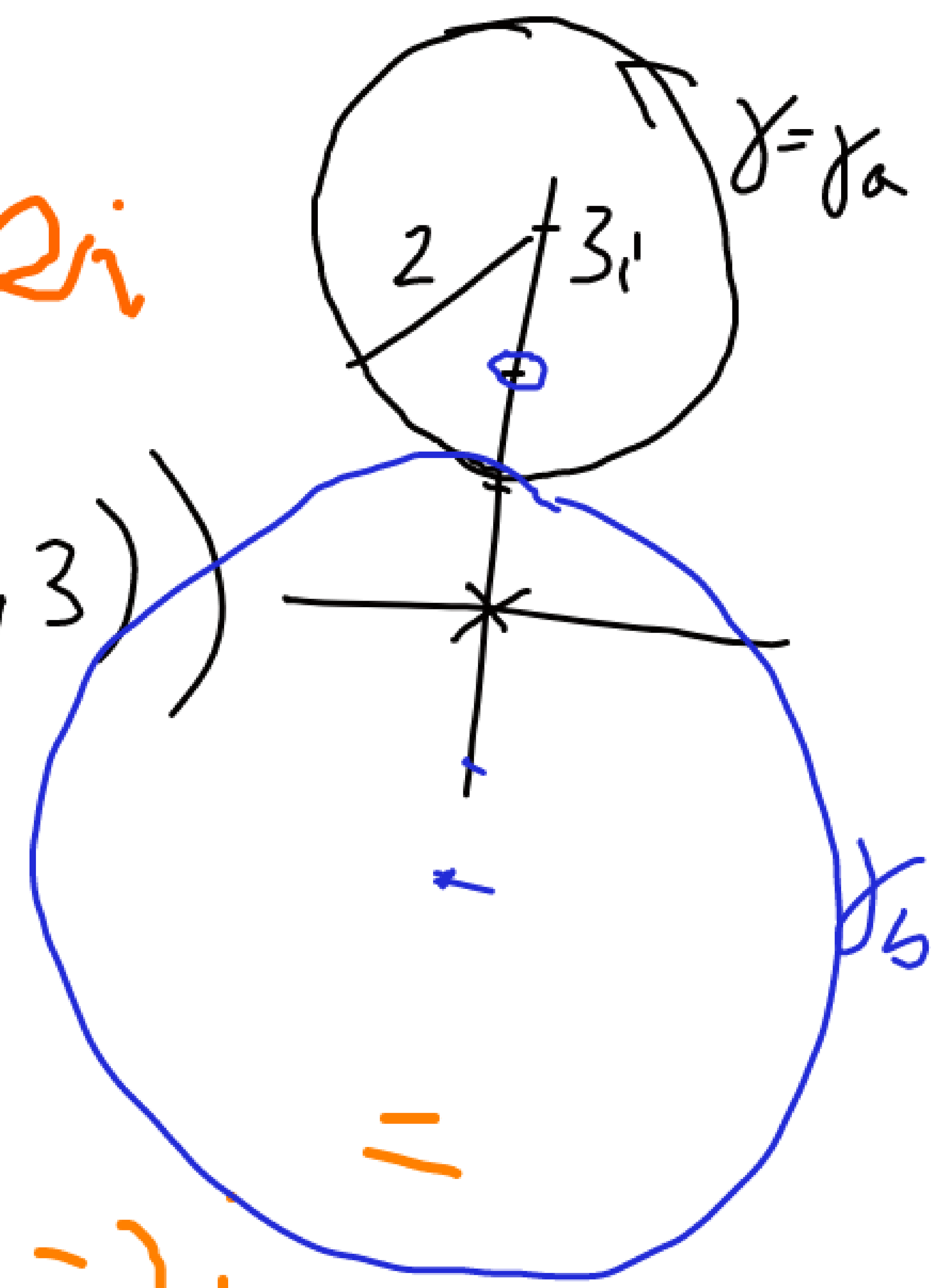
$\Omega \subset \mathbb{C}$ gwiaździsty

$\underset{\text{zamknięta}}{\gamma^*} \subset \Omega$, $f \in H(\Omega)$

a) $$\int_\gamma \frac{e^z}{z(z-2i)}\,dz = \int_\gamma \frac{\frac{e^z}{z}}{z-2i}\,dz = 2\pi i\,\frac{e^z}{z}\Big|_{z=2i}$$

$$= 2\pi i \cdot \frac{e^{2i}}{2i} = \pi e^{2i}$$

$\left(z \mapsto \frac{e^z}{z}\right) \in H(D(3i,3))$



$$\int_\gamma \frac{\sin z}{z^2(z-2i)^3}\,dz = \int_\gamma \frac{\frac{\sin z}{z^2}}{(z-2i)^3}\,dz = \frac{2\pi i}{2}\left(\frac{\sin z}{z^2}\right)''\Big|_{z=2i} =$$

$$\frac{2\pi i}{2}\left(\frac{(z^2-6)\sin z + 4z\cos z}{z^4}\right)\Big|_{z=2i} =$$

b) $$\int_\gamma \frac{e^z}{z(z-2i)}\,dz = \int_\gamma \frac{\frac{e^z}{z-2i}}{z}\,dz = 2\pi i\,\frac{e^z}{z-2i}\Big|_{z=0} =$$

$\in H(D(-2i,4))$

$$2\pi i \cdot \left(\frac{1}{-2i}\right) = -\pi$$