67 $\quad f \in H(D(a,r))$

a – m-krotnym zerem $f \iff f(a) = f'(a) = \ldots = f^{(m-1)}(a) = 0,\ f^{(m)}(a) \neq 0$

$(\Rightarrow)$ $\cancel{f(z) = \sum_{n=0}^{\infty} b_n (z-a)^n, \ z \in D(a,r)}$

$$f(z) = (z-a)^m g(z) = \quad , \text{gdzie } g \in H(D(a,r)),\ g(a) \neq 0$$

$$\underset{z \in D(a,r)}{=} (z-a)^m \sum_{n=0}^{\infty} b_n (z-a)^n = \sum_{n=m}^{\infty} b_{n-m} (z-a)^n, \quad z \in D(a,r)$$

$$\Rightarrow f(a) = 0,\ f'(a) = 0, \ldots, f^{(m-1)}(a) = 0,\ f^{(m)}(a) = m!\, b_0 = m!\, g(a) \neq 0$$

$(\Leftarrow)$ $f(z) = \sum_{n=0}^{\infty} c_n (z-a)^n, \ z \in D(a,r)$

skoro $f^{(k)}(a) = 0$, $k < m$, to $c_k = 0$ dla $k < m$, czyli

$$f(z) = \sum_{n=m}^{\infty} c_n (z-a)^n = (z-a)^m \cdot \underbrace{\sum_{n=0}^{\infty} c_{n+m} (z-a)^n}_{\in H(D(a,r)),}$$

w punkcie a jest $= c_m = \dfrac{f^{(m)}(a)}{m!} \neq 0$