

$$b) f(z) = \frac{z}{z^2 + 1}$$

$$\Omega = \mathbb{C} \setminus \overline{D(0, 1)}$$

$$\int_{\gamma} \frac{z}{z^2 + 1} dz = \int_{\gamma} \frac{\frac{1}{2}}{z+i} dz + \int_{\gamma} \frac{\frac{1}{2}}{z-i} dz =$$

$$\left\{ \begin{aligned} \frac{z}{z^2 + 1} &= \frac{A}{z+i} + \frac{B}{z-i} = \frac{A(z-i) + B(z+i)}{z^2 + 1} \end{aligned} \right.$$

$$A(z-i) + B(z+i) = z$$

$$z=i: \quad 2iB = i \quad B = \frac{1}{2}$$

$$z=-i: \quad -2iA = -i \quad A = \frac{1}{2}$$

$$= 2\pi i \cdot \frac{1}{2} \left(\text{Ind}_{\gamma}(-i) + \text{Ind}_{\gamma}(i) \right) = 2\pi i \neq 0$$

f nie ma funkcji pierwotnej w Ω .

