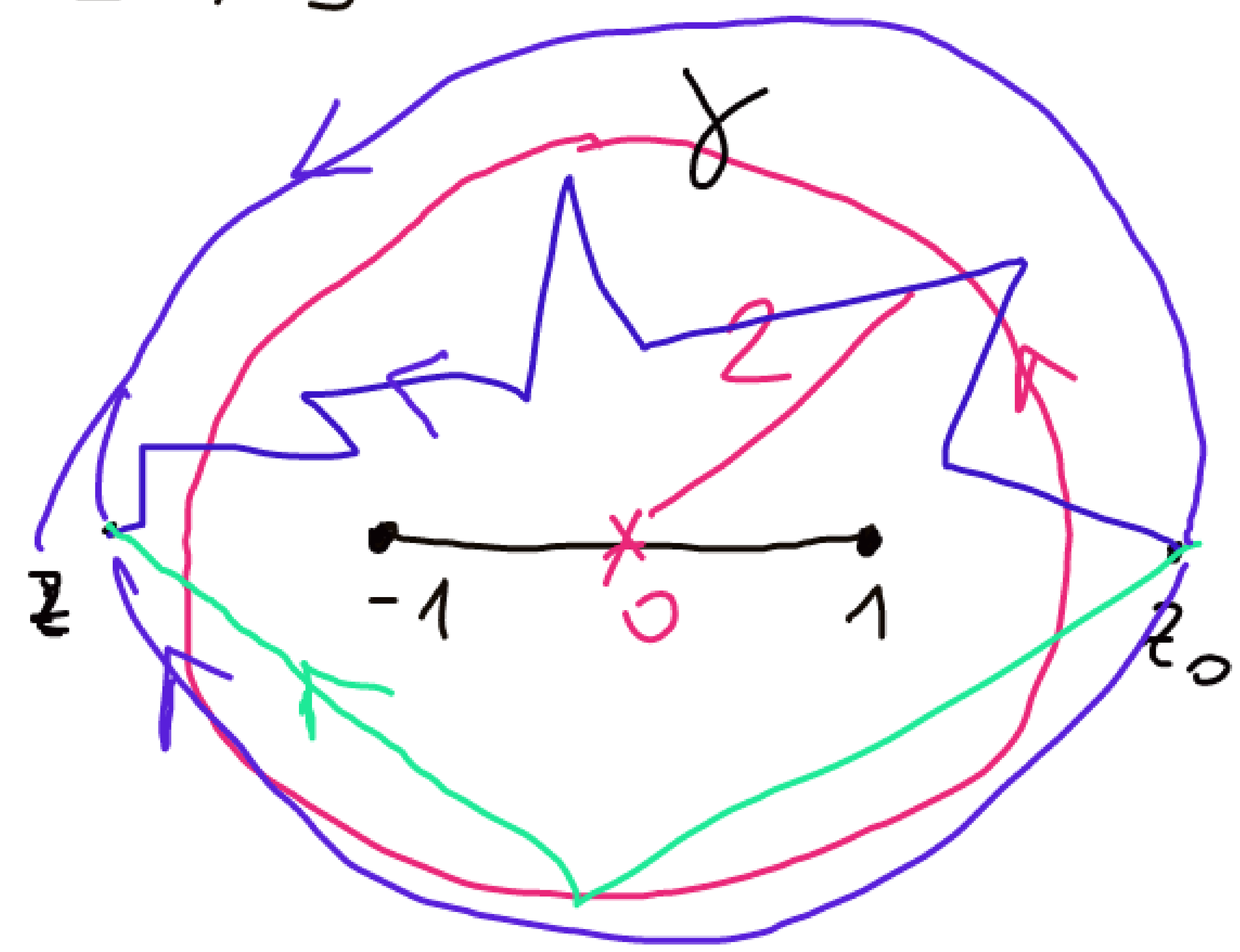
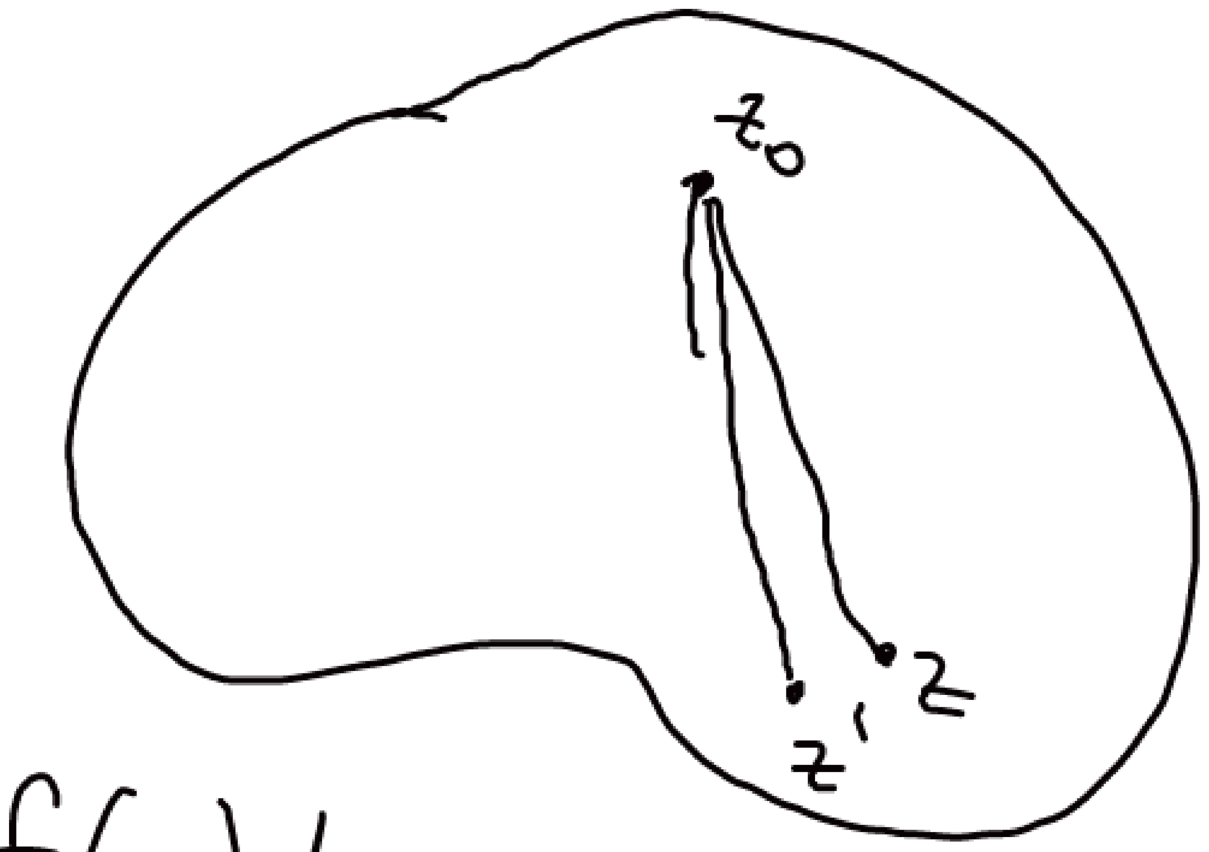
$$f(z) = \frac{1}{z^2-1} = \underbrace{\frac{\frac{1}{2}}{z-1}} + \underbrace{\frac{-\frac{1}{2}}{z+1}}$$

$$\Omega = \mathbb{C} \setminus [-1,1]$$



$$\int_\gamma f(z)\,dz = \int_\gamma \frac{\frac{1}{2}}{z-1}\,dz + \int_\gamma \frac{-\frac{1}{2}}{z+1}\,dz =$$

$$= \frac{1}{2}\overset{2\pi i}{\vee}\underbrace{\mathrm{Ind}_\gamma(1)}_{=1} - \frac{1}{2}\cdot 2\pi i\,\underbrace{\mathrm{Ind}_\gamma(-1)}_{=1} = 0$$



f. pierwotna

$$\frac{1}{z} \rightsquigarrow \mathrm{Log} \quad (w \quad \mathbb{C}\setminus(-\infty,0])$$

$$\frac{1}{z-1} \qquad \mathrm{Log}(z-1) \leftarrow \mathbb{C}\setminus(-\infty,1]$$

$$\frac{1}{z+1} \qquad \mathrm{Log}(z+1) \qquad \mathbb{C}\setminus(-\infty,-1]$$

$$F(z) = \int_{\langle z_0, z\rangle} f(w)\,dw$$

$$f = \frac{1}{2}\left(\frac{1}{z-1} - \frac{1}{z+1}\right) \overset{?}{\rightsquigarrow} \boxed{\frac{1}{2}\,\mathrm{Log}\,\frac{z-1}{z+1}} \text{ - f. pierwotna?}$$

$$\left(\frac{1}{2}\operatorname{Log}\frac{z-1}{z+1}\right)' = \frac{1}{2}\,\frac{1}{\frac{z-1}{z+1}}\cdot\left(\frac{z-1}{z+1}\right)' = \frac{1}{2}\,\frac{1\cdot(z+1)-(z-1)\cdot 1}{\frac{z-1}{z+1}\,(z+1)^2} =$$

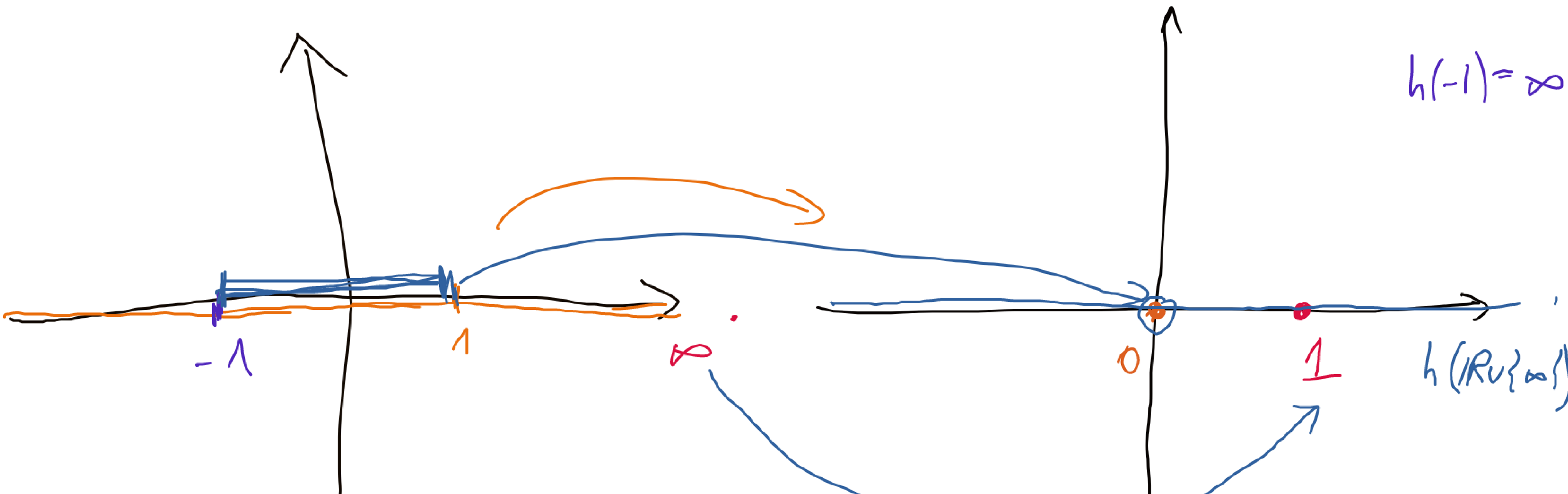
$$= \frac{1}{2}\,\frac{2}{(z-1)(z+1)} = \frac{1}{z^2-1} = f(z)$$

↑ dla takich z, dla których: $z \neq -1$

$$h(z) = \frac{z-1}{\underbrace{z+1}} \in \mathbb{C} \setminus (-\infty, 0]$$

np. dla $z \in \mathbb{C} \setminus [-1,1]$

Na co będzie przekształcony odcinek $[-1,1]$ przez homografię h?



Na $(-\infty, 0] \cup \{\infty\}$.

$$h(\mathbb{C}\setminus[-1,1]) = \mathbb{C} \setminus \left((-\infty,0] \cup \{1\}\right)$$