$$f(z) = \frac{z - \pi i}{1 + e^z}$$

$$e^z = -1 \iff z = \pi i + 2k\pi i \quad (k \in \mathbb{Z})$$

- $$\lim_{z \to \pi i} \frac{z - \pi i}{1 + e^z} = \lim_{z \to \pi i} \frac{1}{\frac{e^z + 1}{z - \pi i}} = \lim_{z \to \pi i} \frac{1}{\frac{e^z - e^{\pi i}}{z - \pi i}} = \frac{1}{(e^z)'\big|_{z = \pi i}} = \frac{1}{e^{\pi i}} = -1$$

osobliwość pozorna

- $z_k = \pi i + 2k\pi i$, $k \neq 0, k \in \mathbb{Z}$

$$\left.\begin{array}{l} 1 + e^z \text{ ma w } z_k \text{ zero rzędu } 1 \\ z - \pi i \text{ jest } \neq 0 \text{ w } z_k \end{array}\right\} \Rightarrow \left|\frac{z - \pi i}{e^z + 1}\right| \to \infty, \text{ f ma biegun w } z_k$$

(rzędu 1 z 77-78)