

$$f, g \in H(\mathbb{C}) \quad |f(z)| \leq |g(z)| \quad \forall z \in \mathbb{C}$$

Zet. $g \neq 0$.

Niech $h(z) = \frac{f(z)}{g(z)}$

$$\lim_{z \rightarrow a} \underbrace{(z-a)}_0 \cdot \underbrace{\frac{f(z)}{g(z)}}_{|f| \leq |g|} = 0 \Rightarrow \text{wszystkie osobliwosci } h(z) \text{ pozorne}$$

$< \infty$

$$|h(z)| = \frac{|f(z)|}{|g(z)|} \leq 1 \quad \text{z tw. Liouville'a:}$$

$$h = \text{const.} \Rightarrow \frac{f(z)}{g(z)} = \text{const.} \Rightarrow f(z) = \alpha \cdot g(z)$$

$\alpha \in \mathbb{C}, |\alpha| \leq 1$