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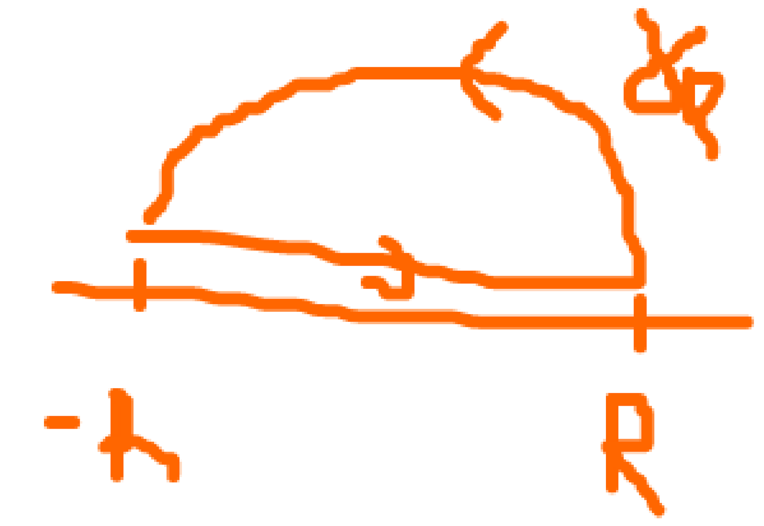
$$\int_{-\infty}^{\infty} \frac{e^{i\xi x}}{1+x^2}\,dx \qquad \xi \in \mathbb{R}$$



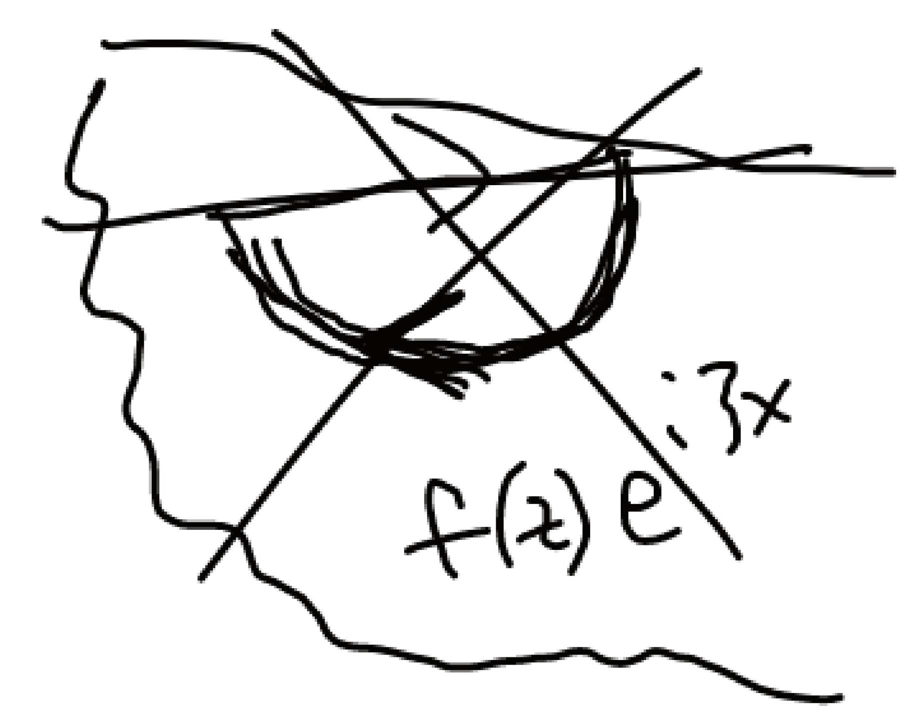
i – biegun

$$\gamma_R = Re^{it},\ t\in[0,\pi]$$

$$\gamma = (-R,R) + \gamma_R$$

$$\int_\gamma f(z)\,dz = 2\pi i\ \mathrm{res}(f,i)\cdot \mathrm{ind}(i)$$

$$\mathrm{res}(f,i) = \lim_{z\to i} \frac{e^{i\xi z}}{z+i} = \frac{e^{-\xi}}{2i}$$



$$\int_{-\infty}^{\infty} f(z)\,dz = \pi e^{-\xi}, \quad \xi>0 \qquad (1)$$

$$\int_{-\infty}^{\infty} f(z)\,dz = \operatorname{arctg} z \Big|_{-\infty}^{\infty} = \pi\cdot e^{0}, \quad \xi=0$$

$$\int_{-\infty}^{\infty} \frac{e^{i\xi z}}{z^2+1}\,dz = \left| x=-z \right| = \int_{-\infty}^{\infty} \frac{e^{-i\xi x}}{(-x)^2+1}\,dx = \int_{-\infty}^{\infty} \frac{e^{i\overbrace{(-\xi)}^{>0}\cdot x}}{x^2+1}\,dx \overset{z\,(1)}{=} \pi e^{-(-\xi)} = \pi e^{-|\xi|}, \quad \xi<0$$