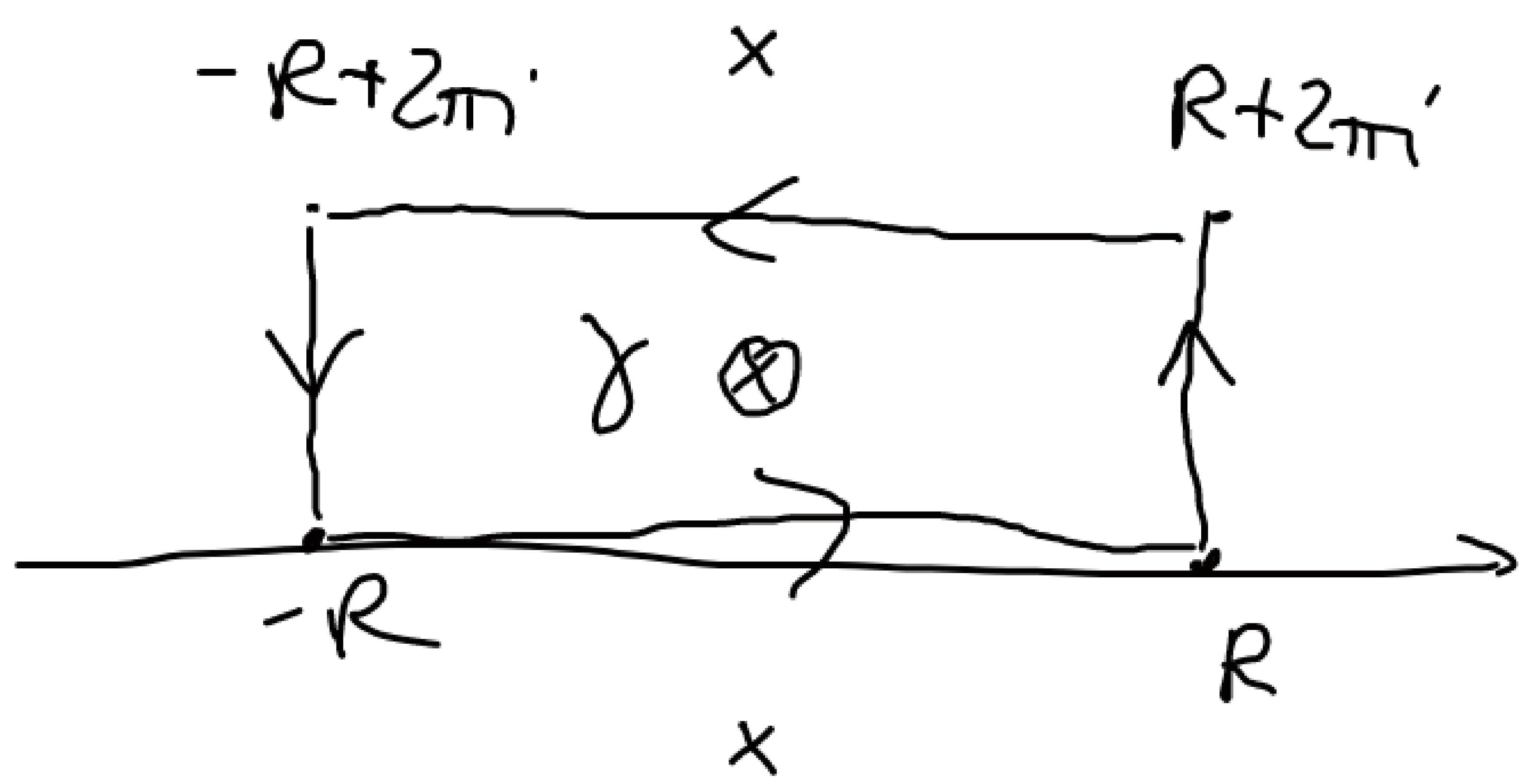
**81e** $\quad a \in (0,1)$

$$\int_{-\infty}^{\infty} \underbrace{\frac{e^{ax}}{e^x+1}}_{f(x)} dx = I$$




$$\int_\gamma f(x)\,dx = 2\pi i \,\mathrm{res}(f, \pi i) =$$

$$= 2\pi i \lim_{x\to\pi i} \frac{e^{ax}}{e^x+1}(x-\pi i) =$$

$$= 2\pi i \lim_{x\to \pi i} \frac{e^{ax}}{\frac{e^x - e^{\pi i}}{x-\pi i}} = 2\pi i \frac{e^{a\pi i}}{(e^x)'\big|_{x=\pi i}} =$$

$$= 2\pi i \frac{e^{a\pi i}}{e^{\pi i}} = -2\pi i\, e^{a\pi i} = I - e^{2\pi i a} I = I(1-e^{2\pi i a})$$

$$e^x+1=0$$

$$e^x=-1 \iff x=\pi i + 2k\pi i, \quad k\in\mathbb{Z}$$

- $$\int_{\langle -R+2\pi i,\, R+2\pi i\rangle} f(x)\,dx = \int_{-R}^{R} \frac{e^{a(t+2\pi i)}}{e^{t+2\pi i}+1}\cdot 1\cdot dt = e^{2\pi i a}\int_{-R}^{R} \frac{e^{at}\,dt}{e^t+1} = e^{2\pi i a}\int_{\langle -R,R\rangle} f(x)\,dx$$

  $(\gamma(t) = t+2\pi i,\ t\in[-R,R])$

- $$\left|\int_{\langle -R,\,-R+2\pi i\rangle} f(x)\,dx\right| = \left|\int_0^{2\pi}\frac{e^{a(-R+it)}\cdot i}{e^{-R+it}+1}\,dt\right| \le \int_0^{2\pi}\frac{e^{-aR}}{|\underbrace{e^{-R+it}}_{|\cdot|=e^{-R}}+1|}\,dt \le$$

  $$\le \int_0^{2\pi}\frac{e^{-aR}}{1-e^{-R}}\,dt = 2\pi\frac{e^{-aR}}{1-\underbrace{e^{-R}}_{\to 0}} \xrightarrow[R\to\infty]{} 0$$

- $$\left|\int_{\langle R,\,R+2\pi i\rangle} f(x)\,dx\right| = \left|\int_0^{2\pi}\frac{e^{a(R+it)}\, i}{e^{R+it}+1}\,dt\right| \le \int_0^{2\pi}\frac{e^{aR}}{|\underbrace{e^{R+it}}_{|\cdot|=e^{R}}+1|}\,dt \le$$

  $$\le \int_0^{2\pi}\frac{e^{aR}}{e^R-1}\,dt = 2\pi\frac{1}{\underbrace{e^{R(1-a)}}_{\to\infty} - \underbrace{e^{-aR}}_{\to 0}} \xrightarrow[R\to\infty]{} 0$$