84)

$$f(z)=\frac{1-e^{2iz}}{z^2}$$

osobliwość jest tylko w 0

Z tw. Cauchy'ego: $$\int_\gamma f(z)\,dz = 0.$$

$$\mathrm{Re}\left(\int_{-R}^{-r}+\int_r^R\right)f(z)\,dz = \mathrm{Re}\left(\int_{-R}^{-r}+\int_r^R\right)\frac{1-\cos 2z - i\sin 2z}{z^2}\,dz$$

$$=\left(\int_{-R}^{-r}+\int_r^R\right)\frac{2\sin^2 z}{z^2}\,dz \xrightarrow[R\to\infty]{r\to 0} 4\int_0^\infty \frac{\sin^2 z}{z^2}\,dz$$

84 cd

przeciwna orientacja $\curvearrowright \gamma_r$

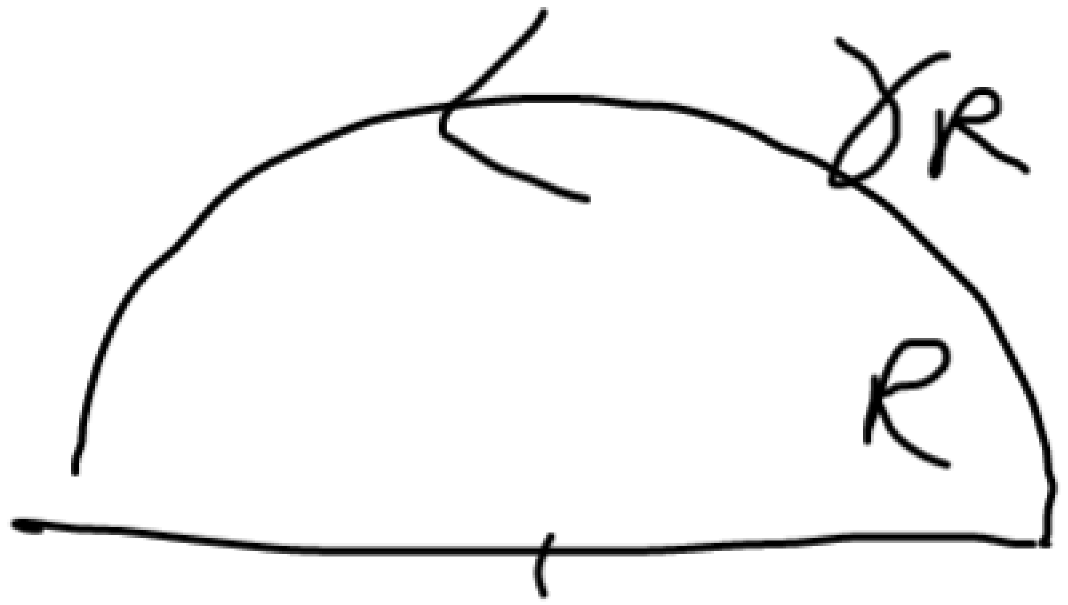
$$\lim_{r\to 0}\int_{\gamma_r} \underbrace{\frac{1-e^{2iz}}{z^2}}_{f(z)}\,dz \underset{83}{=} i\cdot(-2i)\cdot\pi\cdot(-1) = -2\pi$$

$$f\in C(\mathbb{C}\setminus\{0\}) \qquad \lim_{z\to 0} f(z)\cdot z = \lim_{z\to 0}\frac{1-e^{2iz}}{z-0} = -\left(e^{2iz}\right)'\Big|_{z=0} =$$

$$= -2i\,e^{2iz}\Big|_{z=0} = -2i$$

84 cd

$$\int_{\gamma_R} \frac{1-e^{2iz}}{z^2}\,dz =$$



$\gamma_R(t) = Re^{it}$,

$t \in [0, \pi]$

$$= \underbrace{\int_{\gamma_R} \frac{1}{z^2}\,dz}_{\to 0} - \underbrace{\int_{\gamma_R} \frac{e^{2iz}}{z^2}\,dz}_{\to 0}$$

bo st. mian. = st. licz. + 2

z lematu Jordana

8Ycd.

$$0 = \int_\gamma f(z)\,dz \xrightarrow[\substack{r\to 0\\ R\to\infty}]{} 4\cdot\int_0^\infty \frac{\sin^2 x}{x^2}\,dx - 2\pi$$

$\boxtimes$