87 $\quad f_0=f_1=1 \qquad f_{n+2}=f_{n+1}+f_n,$

a) $F(z)=\underbrace{\sum_{n=0}^{\infty} f_n z^n}$, $|z|<\frac{1}{2}$ — było

$1-z-z^2=0$

$\Delta=5$

$z_{1,2}=\frac{1\pm\sqrt{5}}{-2}=\frac{-1\mp\sqrt{5}}{2}$

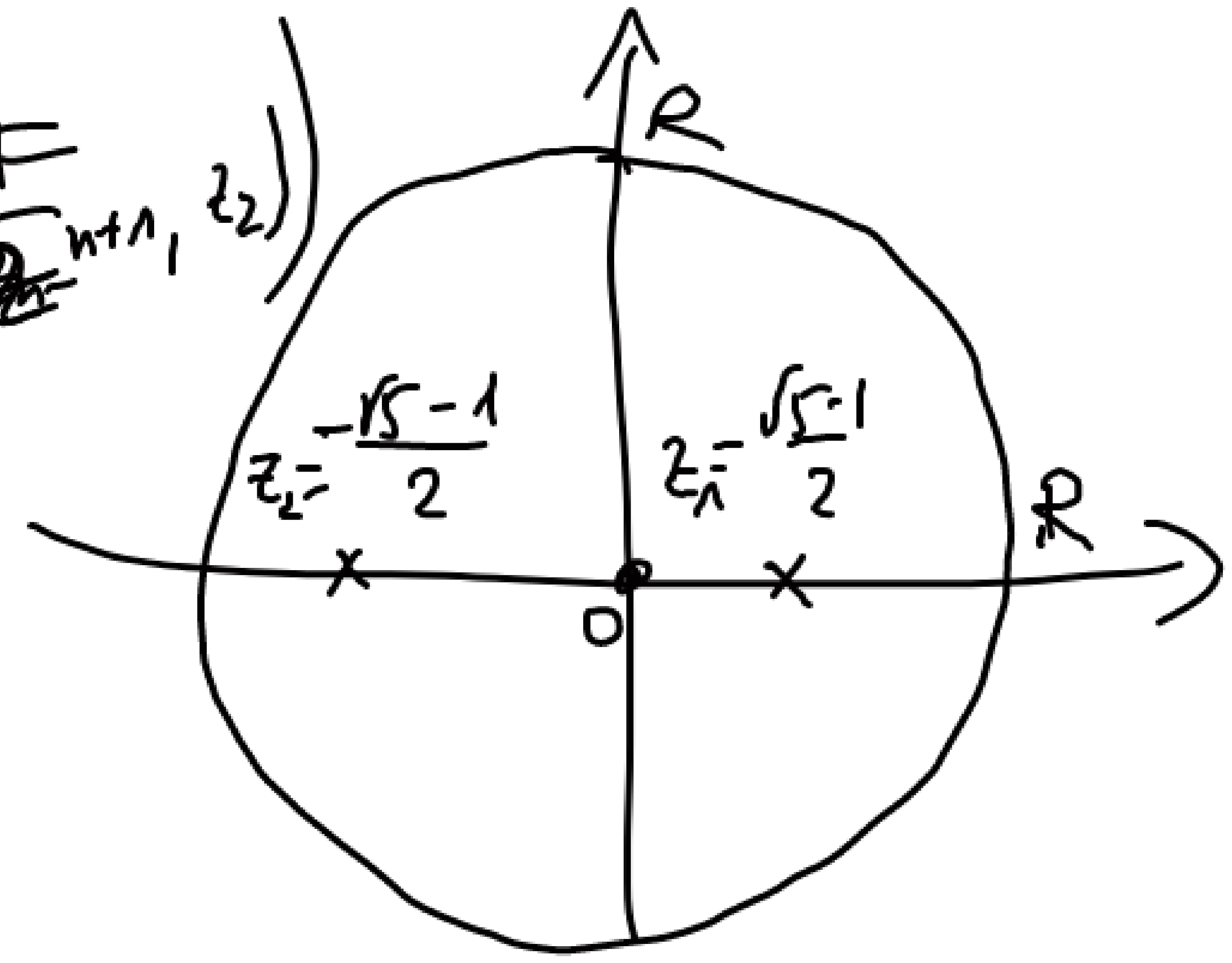
$F(z)=\frac{1}{1-z-z^2}$ tam gdzie szereg F jest zbieżny

$$\frac{-1}{(z-z_1)(z-z_2)}=F(z)=\frac{-1}{\left(z+\left(\frac{1+\sqrt{5}}{2}\right)\right)\left(z+\frac{1-\sqrt{5}}{2}\right)}$$

c)

$$\operatorname{res}\Big(\underbrace{\frac{F(z)}{z^{n+1}}}_{\frac{(g\cdot z^{n+1})^{(n)}}{n!}(0)}\,;0\Big)=\operatorname{res}\Big(\underbrace{\frac{f_0}{z^{n+1}}+\frac{f_1}{z^n}+\ldots+\frac{f_n}{z^1}}_{\text{część główna bieguna}}+\underbrace{\frac{f_{n+1}}{1}+f_{n+2}z+f_{n+3}z^2+\ldots}_{\text{f. holomorficzna}};0\Big)=f_n.$$

(79)



$$\underbrace{\int_{C_R}\frac{F(z)}{z^{n+1}}dz}=2\pi i\left(\operatorname{res}\left(\frac{F}{z^{n+1}},0\right)+\operatorname{res}\left(\frac{F}{z^{n+1}},z_1\right)+\operatorname{res}\left(\frac{F}{z^{n+1}},z_2\right)\right)$$

$$=2\pi i\left(f_n+\lim_{z\to z_1}\frac{F(z)(z-z_1)}{z^{n+1}}+\lim_{z\to z_2}\frac{F(z)(z-z_2)}{z^{n+1}}\right)=$$

$$=2\pi i\left(f_n+\frac{-1}{(z_1-z_2)z_1^{n+1}}+\frac{-1}{(z_2-z_1)z_2^{n+1}}\right)$$

dla dużych R
$(R>|z_2|)$

87c-cd.

$$\left|\int_{C_R} \frac{dz}{z^{n+1}(1-z-z^2)}\right| \le \int_{C_R} \frac{|dz|}{\underbrace{|z|^{n+1}}_{=R}\,\underbrace{|1-z-z^2|}_{\ge R^2\cdot\frac{1}{2}}} \le \frac{1}{R^{n+1}}\,\frac{2}{R^2}\cdot 2\pi R \xrightarrow[R\to\infty]{} 0$$

Zatem

$$0 = f_n + \frac{-1}{(z_1-z_2)(z_1)^{n+1}} + \frac{-1}{(z_2-z_1)\,z_2^{n+1}}$$

$$f_n = \frac{1}{(z_1-z_2)\,z_1^{n+1}} + \frac{1}{(z_2-z_1)\,z_2^{n+1}} =$$

$$z_1 = \frac{\sqrt5-1}{2} \qquad z_2 = \frac{-\sqrt5-1}{2} \qquad z_1-z_2=\sqrt5$$

$$z_2-z_1 = -\sqrt5$$

$$\frac{1}{z_1} = \frac{2}{\sqrt5-1} = \frac{2(\sqrt5+1)}{4} = \frac{\sqrt5+1}{2}$$

$$\frac{1}{z_2} = \frac{-2}{\sqrt5+1} = \frac{-2(\sqrt5-1)}{4} = \frac{1-\sqrt5}{2}$$

$$= \frac{1}{\sqrt5}\left(\frac{1+\sqrt5}{2}\right)^{n+1} - \frac{1}{\sqrt5}\left(\frac{1-\sqrt5}{2}\right)^{n+1}$$

dla dużych z: $|1-z-z^2| \ge \frac{R^2}{2}$

dlaczego?

$$\left|\frac{1-z-z^2}{z^2}\right| \xrightarrow[|z|=R\to\infty]{} 1$$

$$\left|\frac{1-z-z^2}{z^2}\right| = \left|\frac{1}{z^2}-\frac{1}{z}-1\right|$$

zatem dla dużych R i $|z|=R$:

$$\left|\frac{1-z-z^2}{z^2}\right| \ge \frac{1}{2}$$

$$|1-z-z^2| \ge \frac{1}{2}|z^2| = \frac{1}{2}R^2$$