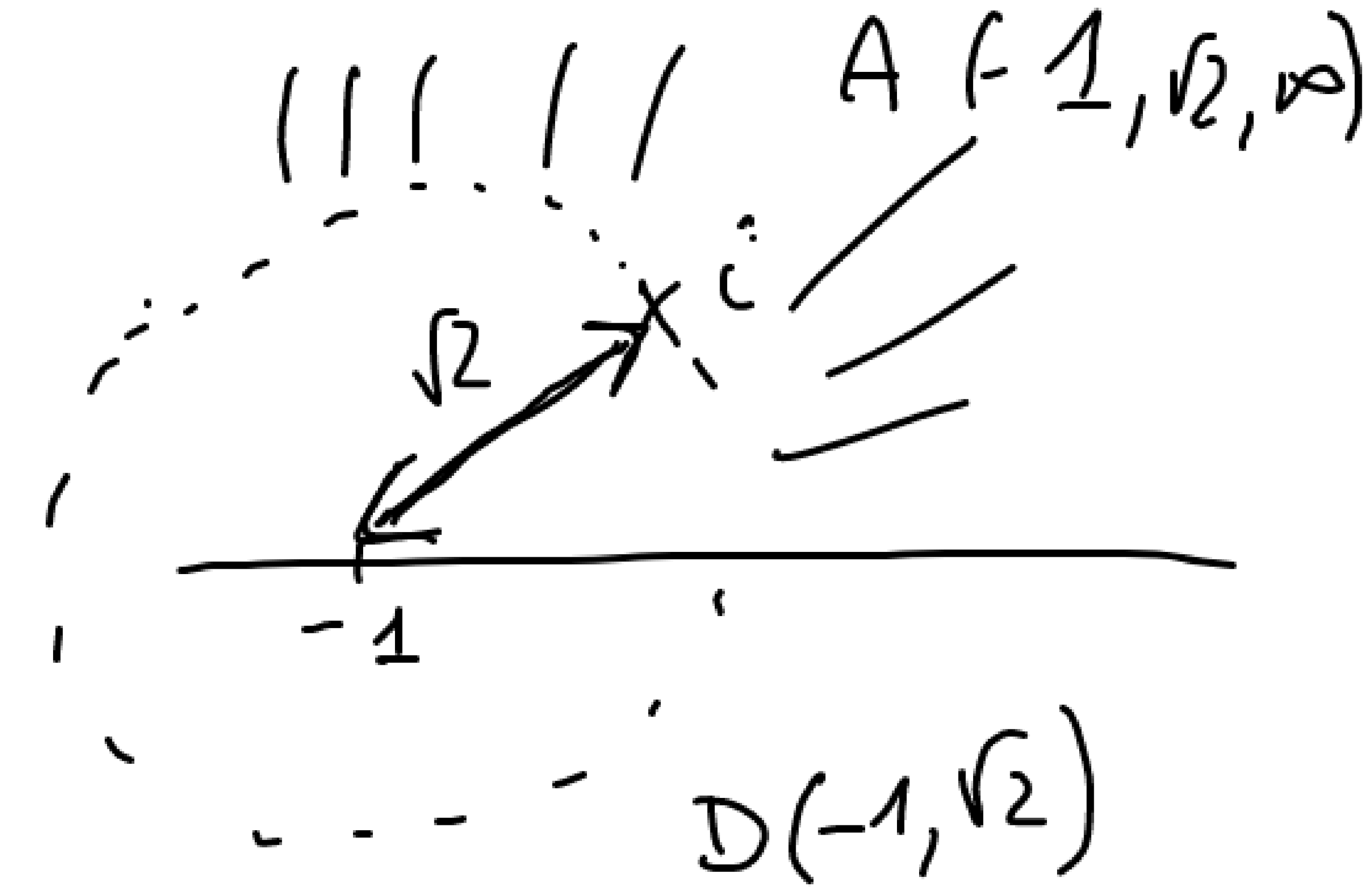
89 – podobne

$A(-1, \sqrt{2}, \infty)$ potęgi $z+1$

$$\frac{1}{z-i} = \frac{1}{\underbrace{(z+1)}_{\substack{\text{ma moduł} \\ >\sqrt{2}}} - \underbrace{(i+1)}_{\text{ma moduł } \sqrt{2}}} =$$

„duże" „małe"



$$= \frac{1}{z+1}\,\frac{1}{1-\underbrace{\frac{i+1}{z+1}}_{\text{ma moduł } <1}} = \frac{1}{z+1}\sum_{n=0}^{\infty}\left(\frac{i+1}{z+1}\right)^n =$$

$$\frac{1}{1-w} = \sum_{n=0}^{\infty} w^n, \quad |w|<1$$

„duże" „małe"

$$= \sum_{n=0}^{\infty}\frac{(i+1)^n}{(z+1)^{n+1}} = \sum_{n=0}^{\infty}(i+1)^n (z+1)^{\overset{k}{-n-1}} = \sum_{k=-\infty}^{-1}(i+1)^{-k-1}(z+1)^k$$

$k = -n-1$

$n = -k-1$

$n = 0, 1, \ldots$

$k = -1, -2, -3, \ldots$

$$\frac{1}{(z-i)^2} = \frac{1}{z-i}\cdot\frac{1}{z-i}$$

j. zbieżność na obszarze zbieżności

$$\boxed{\frac{1}{(z-i)^2}} = \left(\frac{-1}{z-i}\right)' = \left(-\sum_{k=-\infty}^{-1}(i+1)^{-k-1}(z+1)^k\right)' = -\sum_{k=-\infty}^{-1}(i+1)^{-k-1}k(z+1)^{\overset{m}{k-1}} =$$

$m = k-1$

$k = m+1$

$$= \boxed{-\sum_{m=-\infty}^{-2}(i+1)^{-(m+1)-1}(m+1)(z+1)^m}$$

$A(0, \sqrt{2}, \infty)$