89 a) $A(-1, 0, 2)$

// $\frac{1}{z+1}\left(\frac{1}{z-1}\right)$ rozwinąć (inny sposób)

$$\sum_{n=0}^{\infty} w^n = \frac{1}{1-w}, \quad |w|<1$$

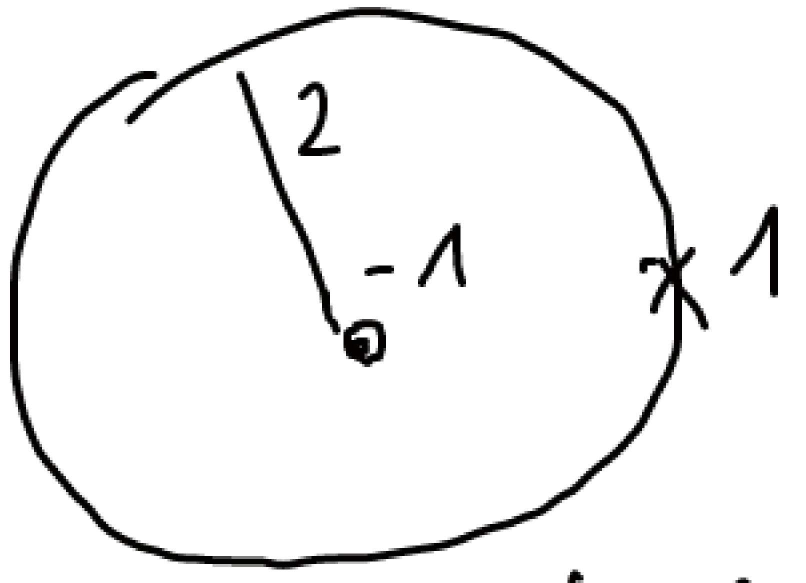
$$f(z) = \frac{1}{z^2-1} = \frac{1}{(z-1)(z+1)} = \frac{1}{2}\,\frac{1}{z-1} - \frac{1}{2}\cdot\frac{1}{z+1} =$$

$$\left|\frac{z+1}{2}\right| < 1 \Leftrightarrow |z+1| < 2$$

$$= \frac{1}{2}\cdot\frac{1}{z+1-2} - \frac{1}{2}\cdot\frac{1}{z+1} = \frac{1}{4}\cdot\frac{1}{-\left(-\frac{z+1}{2}+1\right)} - \frac{1}{2}\cdot\frac{1}{z+1} =$$

$$= -\frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{z+1}{2}\right)^n - \frac{1}{2}\cdot\frac{1}{z+1} = \sum_{n=-\infty}^{\infty} a_n (z+1)^n$$

$$a_n = \begin{cases} 0 & , n=-2,-3,\dots \\ -\frac{1}{2} & , n=-1 \\ -\frac{1}{4}\frac{1}{2^n} & , n=0,1,\dots \end{cases} = \begin{cases} 0 & , n \le -2 \\ -\frac{1}{2^{n+2}} & , n \ge -1 \end{cases}$$

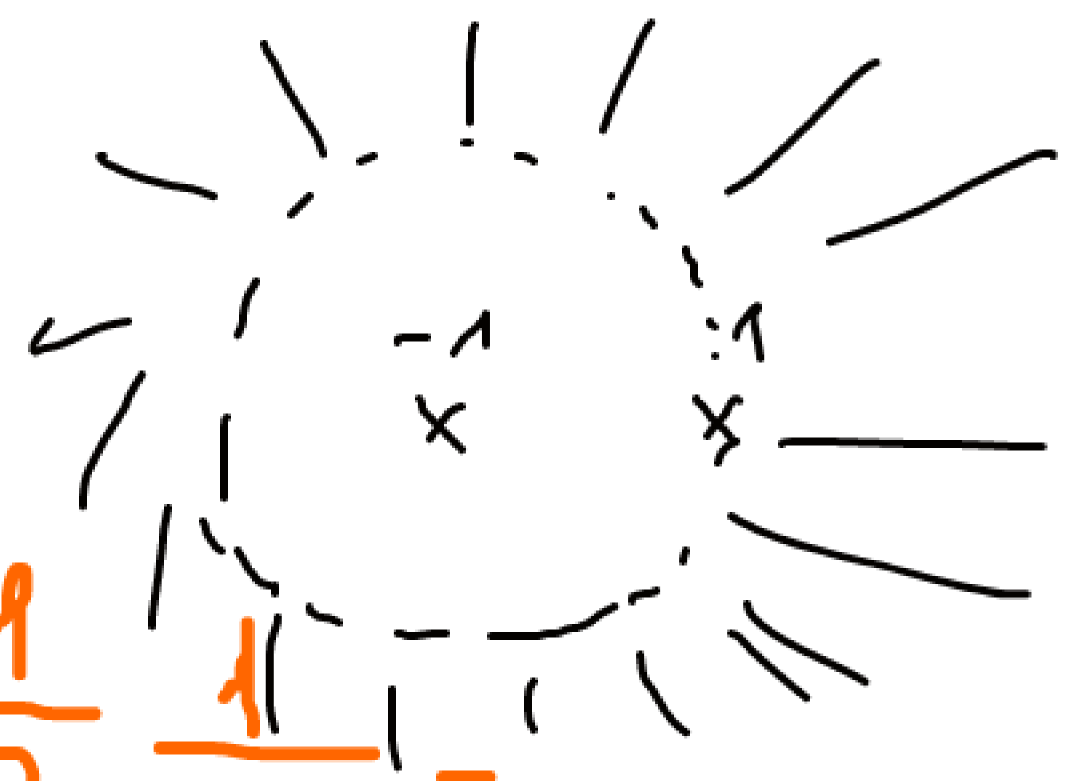


$A(-1,0,2) = D'(-1,2)$

89 a) $f(z)=\frac{1}{z^2-1}=$ $A(-1,2,\infty)$

$$\sum_{n=0}^{\infty} w^n=\frac{1}{1-w},\quad |w|<1$$

mała



$$=\frac{1}{2(z+1)}\cdot\frac{1}{\frac{z-1}{z+1}}-\frac{1}{2}\cdot\frac{1}{z+1}=\frac{1}{2(z+1)}\cdot\frac{1}{1-\frac{2}{z+1}}-\frac{1}{2}\cdot\frac{1}{z+1}=$$

$$=\underbrace{\frac{1}{2(z+1)}\sum_{n=0}^{\infty}\left(\frac{2}{z+1}\right)^n}_{-n-1}-\frac{1}{2}\underbrace{\frac{1}{z+1}}_{\text{"}-1\text{"}}$$

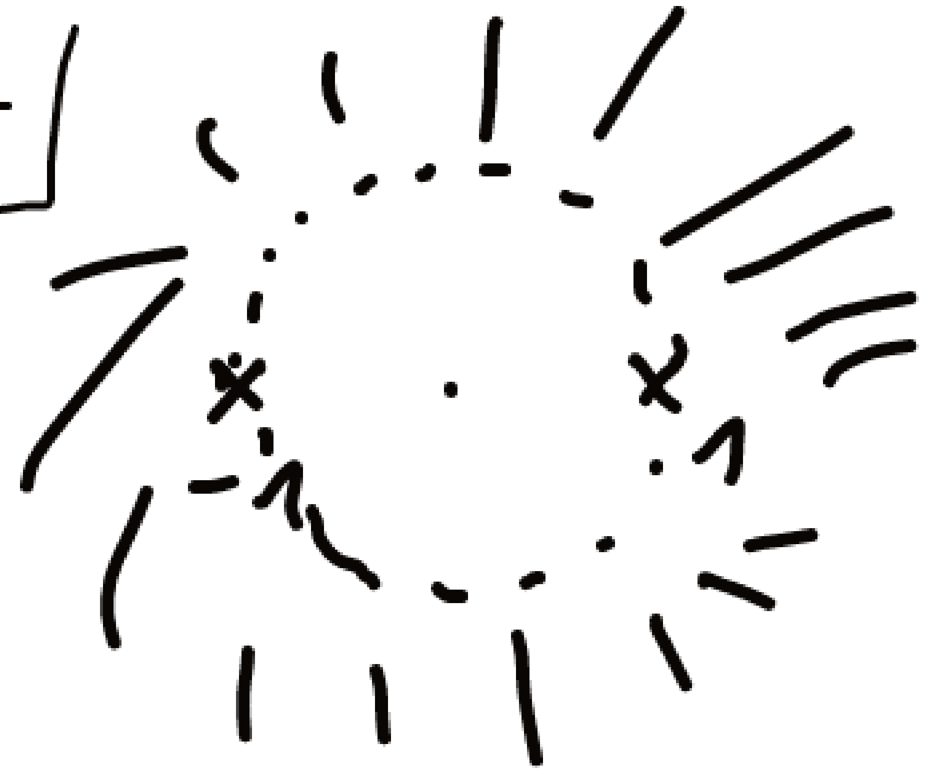
$$\left|\frac{2}{z+1}\right|<1$$

$$\frac{1}{1-\underbrace{\frac{z+1}{2}}_{\text{duże}}}=\frac{1}{\frac{z+1}{2}\left(\frac{2}{z+1}-1\right)}=\frac{-1}{\frac{z+1}{2}\left(1-\underbrace{\frac{2}{z+1}}_{\text{małe}}\right)}=\frac{-1}{\frac{z+1}{2}}\sum_{n=0}^{\infty}\left(\frac{2}{z+1}\right)^n$$

89a-iii

$$f(z)=\frac{1}{z^2-1}=\frac{1}{2}\,\frac{1}{z-1}-\frac{1}{2}\,\frac{1}{z+1}=$$

$$\boxed{\sum_{n=0}^{\infty} w^n=\frac{1}{1-w},\quad |w|<1}$$



$$=\frac{1}{2z}\Big(\frac{1}{1-\underbrace{\tfrac{1}{z}}_{\text{na moduł } <1}}-\frac{1}{1+\frac{1}{z}}\Big)=$$

$$=\frac{1}{2z}\Big(\sum_{n=0}^{\infty}\Big(\frac{1}{z}\Big)^n-\sum_{n=0}^{\infty}\Big(\frac{-1}{z}\Big)^n\Big)=$$

$$=\frac{1}{2z}\sum_{n=0}^{\infty}\frac{1-(-1)^n}{z^n}=\frac{1}{2z}\sum_{k=0}^{\infty}\frac{1-(-1)^{2k+1}}{z^{2k+1}}=\frac{1}{2z}\sum_{k=0}^{\infty}\frac{2}{z^{2k+1}}=$$

$$\left|\frac{1}{z}\right|<1$$

$$=\sum_{k=0}^{\infty}\frac{1}{z^{2k+2}}$$

Inny sposób.

$$\frac{1}{z^2-1}=\frac{-1}{z^2}\,\frac{1}{1-\left(\frac{1}{z^2}\right)}=\frac{-1}{z^2}\sum_{n=0}^{\infty}\Big(\frac{1}{z^2}\Big)^n$$