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$$f(z) = \underline{\exp\left(z+\frac{1}{z}\right)} = e^{z}\cdot e^{\frac{1}{z}} = \sum_{n=0}^{\infty}\frac{z^n}{n!}\cdot\sum_{n=0}^{\infty}\frac{z^{-n}}{n!} \overset{?}{=}$$

$$= \left(1 + z + \frac{z^2}{2!} + \dots\right)\left(1 + \frac{1}{z} + \frac{1}{z^2\, 2!} + \dots\right) =$$

$$= \left(1 + 1 + \frac{1}{(2!)^2} + \frac{1}{(3!)^2} + \dots\right) + \left(1 + \frac{1}{2!\,1!} + \frac{1}{3!\,2!} + \dots\right)\left(z + \frac{1}{z}\right) +$$

$$+ \left(\frac{1}{2!} + \frac{1}{3!\,1!} + \frac{1}{4!\,2!} + \dots\right)\left(z^2 + \frac{1}{z^2}\right) + \dots$$

Z drugiej strony $f(z) = \sum_{n=-\infty}^{\infty} a_n z^n$, $a_n = \frac{1}{2\pi i}\int_\gamma \frac{f(z)}{z^{n+1}}\,dz =$

$$= \frac{1}{2\pi i}\int_0^{2\pi} \exp(\underbrace{e^{it} + e^{-it}}_{2\cos t})\, e^{-it(n+\cancel{1})}\, i\cancel{e^{it}}\, dt \qquad \gamma(t) = e^{it},\ t\in[0,2\pi]$$

$$a_n = \frac{1}{2\pi}\int_0^{2\pi} e^{2\cos t}\, e^{-int}\,dt \rightarrow \frac{a_n + a_{-n}}{2} = \frac{1}{2\pi}\int_0^{2\pi} e^{2\cos t}\cos(nt)\,dt$$

? wynika z bezwzględnej zbieżności obu szeregów

tw. Fubiniego $\qquad \sum|f_n(z)|\quad \sum_m |g_m(z)| < \infty$

$$\underline{\sum_{n=1}^{\infty} f_n(z)\cdot\sum_{m=1}^{\infty} g_m(z)} = \underline{\sum_{n=1}^{\infty}}\left(\sum_{m=1}^{\infty} \underbrace{f_n(z)\, g_m(z)}_{h_z(n,m)}\right) = \sum_{(n,m)\in\{1,\dots,\infty\}^2} f_n(z)\, g_m(z)$$