

92)

$$\begin{aligned} z \Gamma(z) &= \lim_{n \rightarrow \infty} \frac{n! n^z}{z(z+1)\dots(z+n)} \cdot z = \\ &= \lim_{n \rightarrow \infty} \frac{n! n^{z+1}}{(z+1)\dots(z+n+1)} \cdot \frac{z+n+1}{n} = \\ &= \Gamma(z+1) \end{aligned}$$

$$\begin{aligned} \Gamma(z) \Gamma(1-z) &= -z \Gamma(z) \Gamma(-z) = \\ &= -z \left( z e^{\gamma z} \prod_{k=1}^{\infty} \left( 1 + \frac{z}{k} \right) e^{-\frac{z}{k}} \right)^{-1} \left( -z e^{\gamma z} \prod_{k=1}^{\infty} \left( 1 - \frac{z}{k} \right) e^{\frac{z}{k}} \right)^{-1} \\ &= \left( z \prod_{k=1}^{\infty} \left( 1 - \frac{z^2}{k^2} \right) \right)^{-1} = \frac{\pi}{\sin(\pi z)} \end{aligned}$$

$$\Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{1}{2}\right) = \frac{\pi}{\sin \frac{\pi}{2}} = \pi$$

$$\Rightarrow \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$\gamma = 0,5772\dots$  wie wiadomo, (czy  $\gamma \in \mathbb{Q}$ )