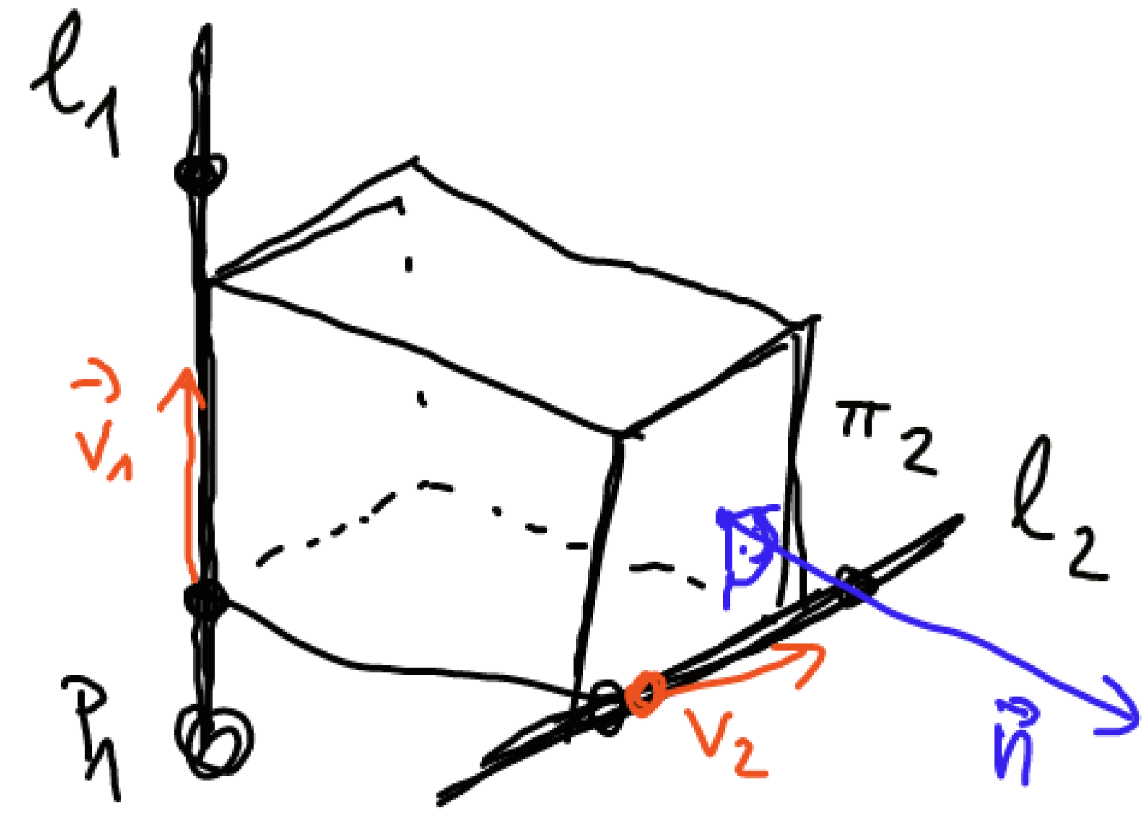
XVI/5

$l_1: \frac{x}{2} = \frac{y+1}{1} = \frac{z+1}{-2}$

$l_2: \frac{x-2}{1} = \frac{y+1}{1} = \frac{z+2}{1}$



Znajdziemy równie płaszczyzny π_2 zawierającej l_2 i $\| l_1$.

$\vec{v}_2 \perp \vec{n}$

$\vec{v}_1 \perp \vec{n}$

np. $P_2 = (2,-1,-2) \in l_2$, a więc $P_2 \in \pi_2$

$\cdot\ \vec{n} \perp \vec{v}_1,\ \vec{n} \perp \vec{v}_2$ $\quad \vec{v}_1 = (2,1,-2),\ \vec{v}_2 = (1,1,1)$

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ab - cd$

$\vec{n} = \vec{v}_1 \times \vec{v}_2 = \left(\begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}, -\begin{vmatrix} 2 & -2 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} \right) = (3,-4,1)$

spr.: $\vec{n} \circ \vec{v}_1 = 3 \cdot 2 + -4 \cdot 1 + 1 \cdot (-2) = 0$

$\vec{n} \circ \vec{v}_2 = 3 - 4 + 1 = 0$

OK

$\pi_2:\ 3(x-2) - 4(y+1) + (z+2) = 0$

$3x - 6 - 4y - 4 + z + 2 = 0$

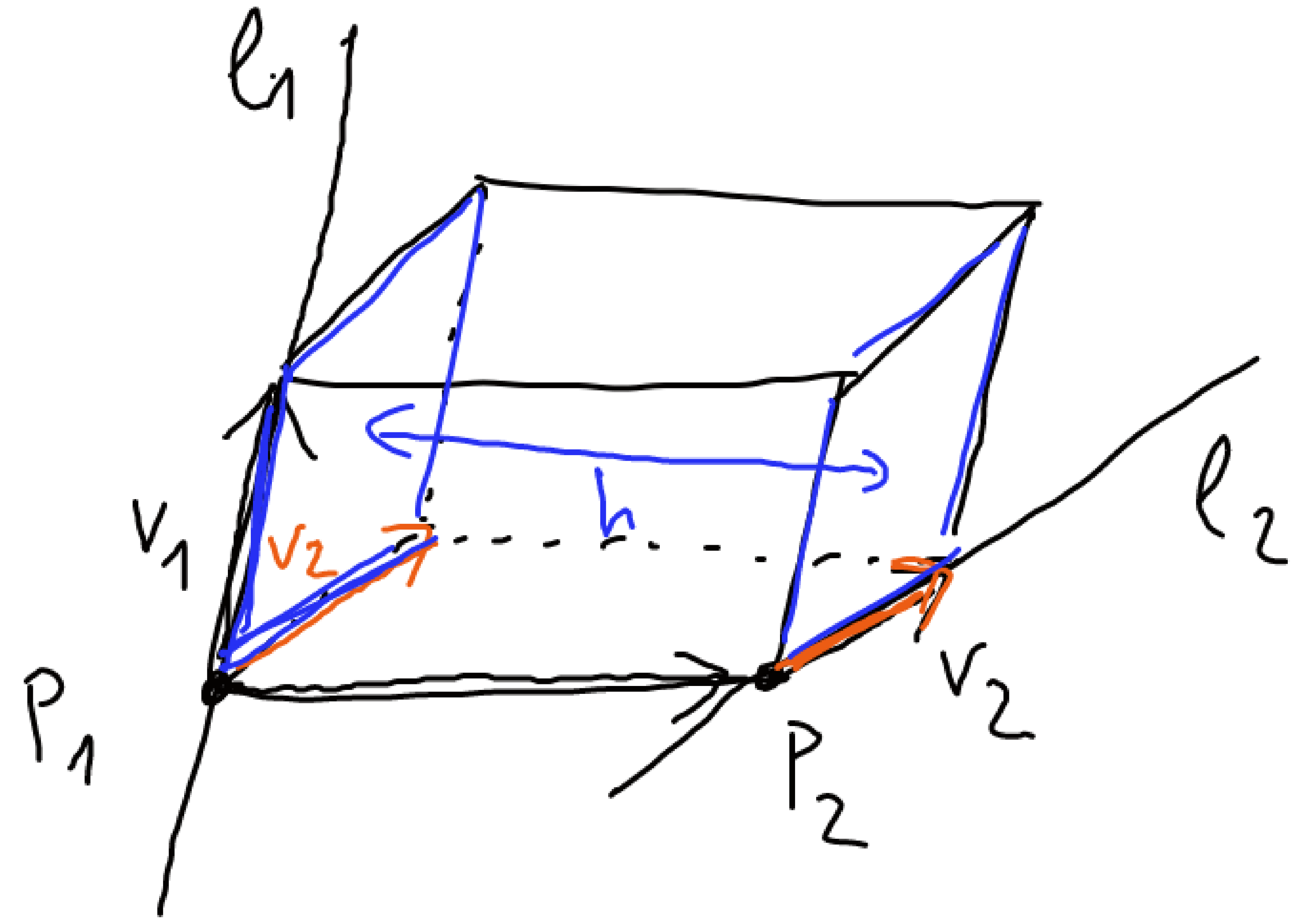
$\boxed{\pi_2: 3x - 4y + z - 8 = 0}$ $\leftarrow$

$d(l_1, l_2) = d(l_1, \pi_2) = d(P_1, \pi_2) = \frac{|3 \cdot 0 - 4 \cdot (-1) + (-1) - 8|}{\sqrt{3^2 + (-4)^2 + 1^2}} = \frac{|-5|}{\sqrt{26}} = \frac{5}{\sqrt{26}}$

$P_1 \in l_1$

np. $P_1 = (0,-1,-1)$ $\leftarrow$

Liczymy objętość równoległościanu na 2 sposoby:



$$V = \left| \left(\vec{v}_1 \times \vec{v}_2\right) \circ \overrightarrow{P_1P_2} \right|$$

$$\parallel$$

$$V = P_{podstawy} \cdot h =$$

$$= \left|\vec{v}_1 \times \vec{v}_2\right| \cdot d(l_1, l_2)$$

$h =$ odl. między podstawami
$= d(l_1, l_2)$

$$d(l_1, l_2) = \frac{\left| \left(\vec{v}_1 \times \vec{v}_2\right) \circ \overrightarrow{P_1P_2} \right|}{\left|\vec{v}_1 \times \vec{v}_2\right|}$$

$l_1 \ni P_1$, $l_1 \parallel \vec{v}_1$

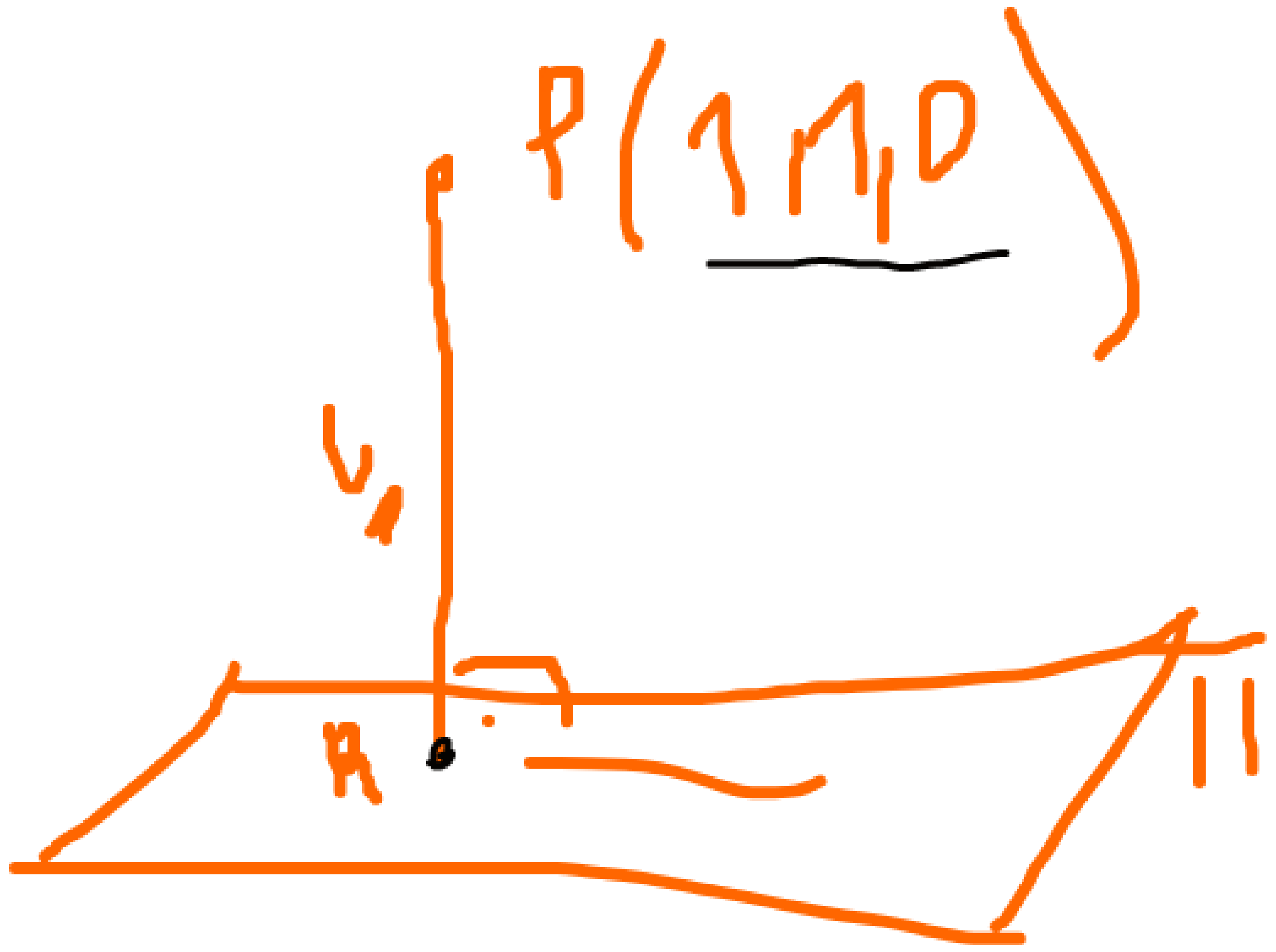
$l_2 \ni P_2$, $l_2 \parallel \vec{v}_2$

Co gdy $\vec{v}_1 \times \vec{v}_2 = \vec{0}$?

Wtedy $l_1 \parallel l_2$, proste nie są skośne i liczymy jak w zadaniu XVI/4.

XVII/1

1



$\vec{n} = (1,-1,3)$

$\Pi: x - y + 3z - 1 = 0$

$(1+t) - (1 -t) + 3(3t) - 1 = 0$

$1 + t - 1 + t + 9t - 1 = 0$

$11t = 1$

$t = \frac{1}{11}$

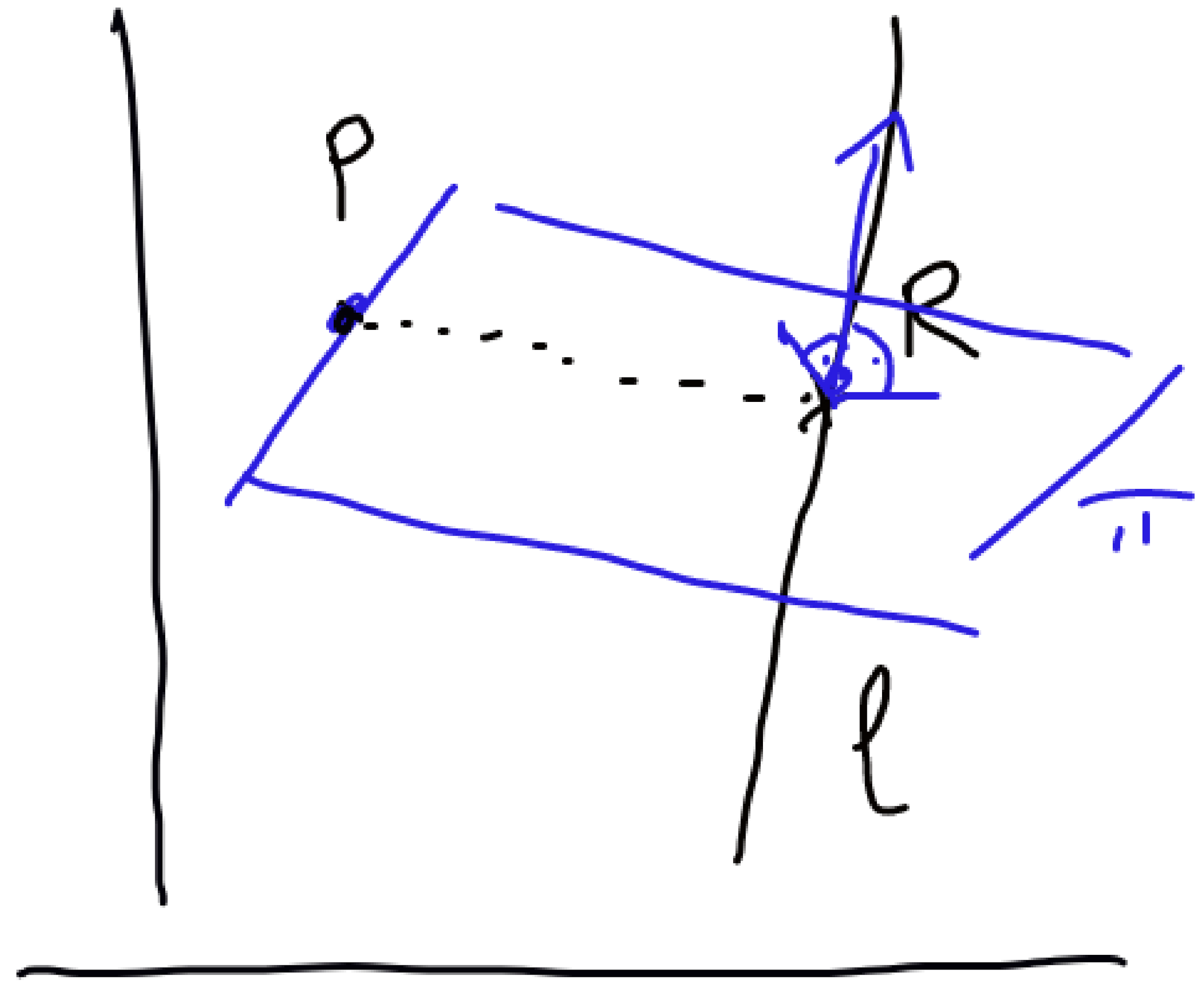
$l_1: \begin{cases} x = 1 + t \\ y = \cancel{1+3t}\ 1 - t \\ z = \cancel{t}\ 3t \end{cases}$

$R\left(1 + \frac{1}{11}, 1 - \frac{1}{11}, \frac{3}{11}\right)$

$= \left(1\frac{1}{11}, \frac{10}{11}, \frac{3}{11}\right).$

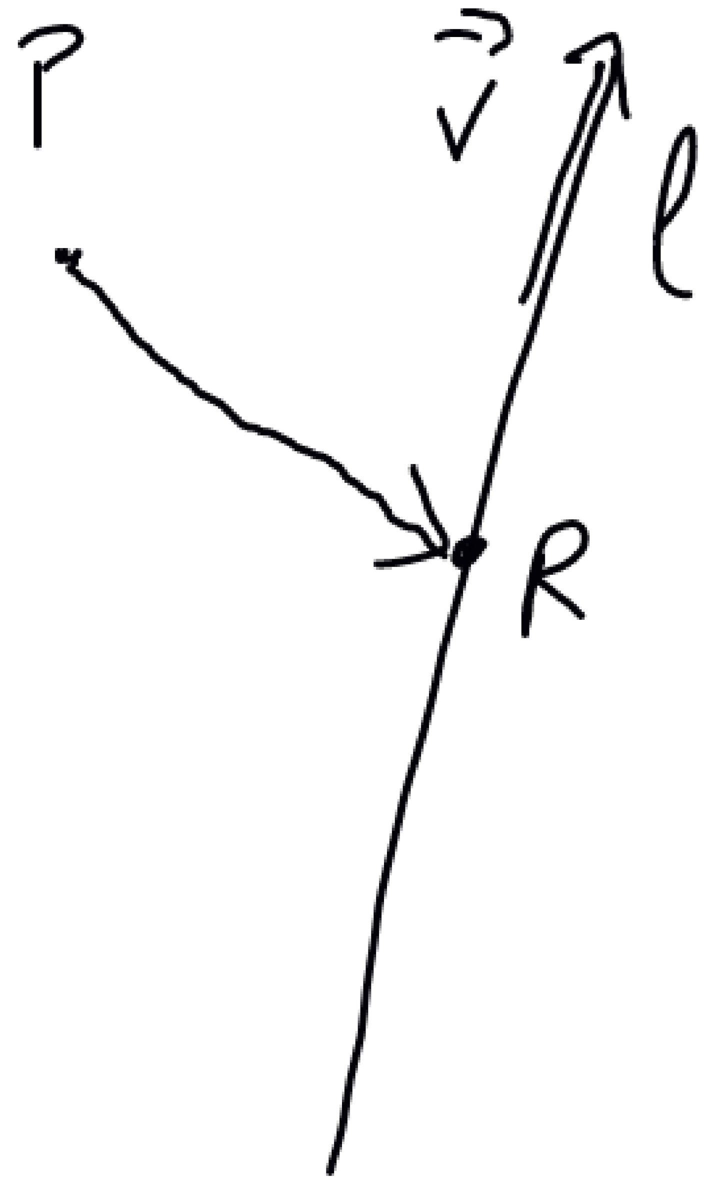
XVII/2 $P=(2,1,1)$

$$\ell: \frac{x-2}{1} = \frac{y}{-2} = \frac{z+1}{1} \to [1,-2,1]$$



$$\begin{cases} x = 2+t \\ y = -2t \\ z = -1+t \end{cases} \qquad \underbrace{R(2+t,-2t,-1+t)}$$

$$\vec{PR} = [2+t-2, -2t-1, -1+t-1] = [t, -2t-1, t-2]$$



$$0 = \vec{PR} \circ [1,-2,1] \to \text{wtedy } \vec{PR} \perp [1,-2,1]$$

$$t\cdot 1 - 2(-2t-1) + (t-2)\cdot 1 = 0$$

$$t + 4t + \cancel{2} + t - \cancel{2} = 0$$

$$6t = 0$$

$$t = 0$$

$$R(2,0,-1)$$

II $\pi: 1(x-2) - 2(y-1) + 1(z-1) = 0 \qquad \pi \cap \ell = \{R\}$

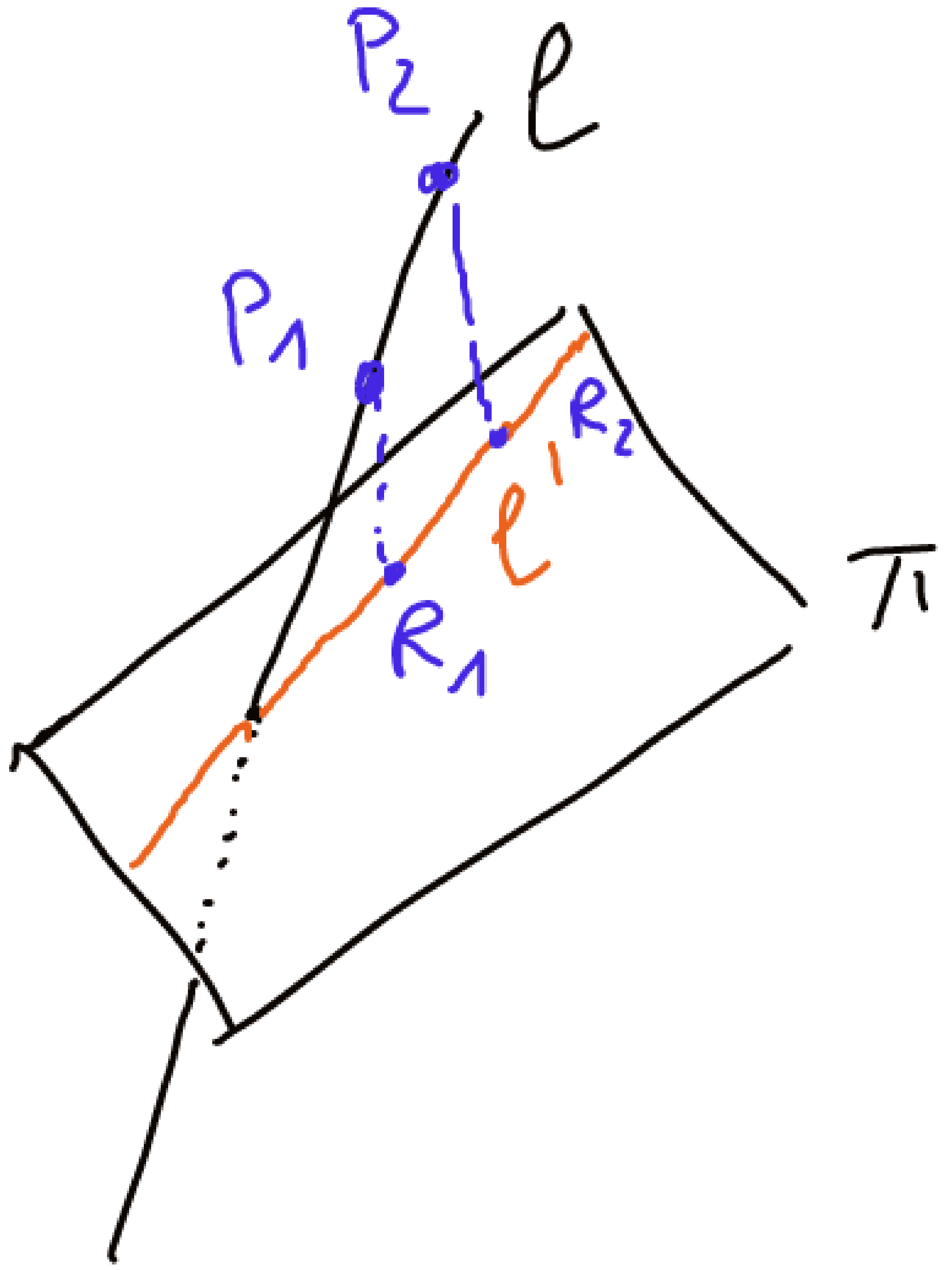
$P \in \pi, \ \pi \perp \ell$

XVI|3

Bierzemy dwa różne punkty $P_1, P_2 \in \ell$

i rzutujemy je na płaszczyznę π,

otrzymujemy punkty R_1, R_2.



1° Jeśli $R_1 = R_2$, to rzutem ℓ na π jest R_1

(pojedynczy punkt)

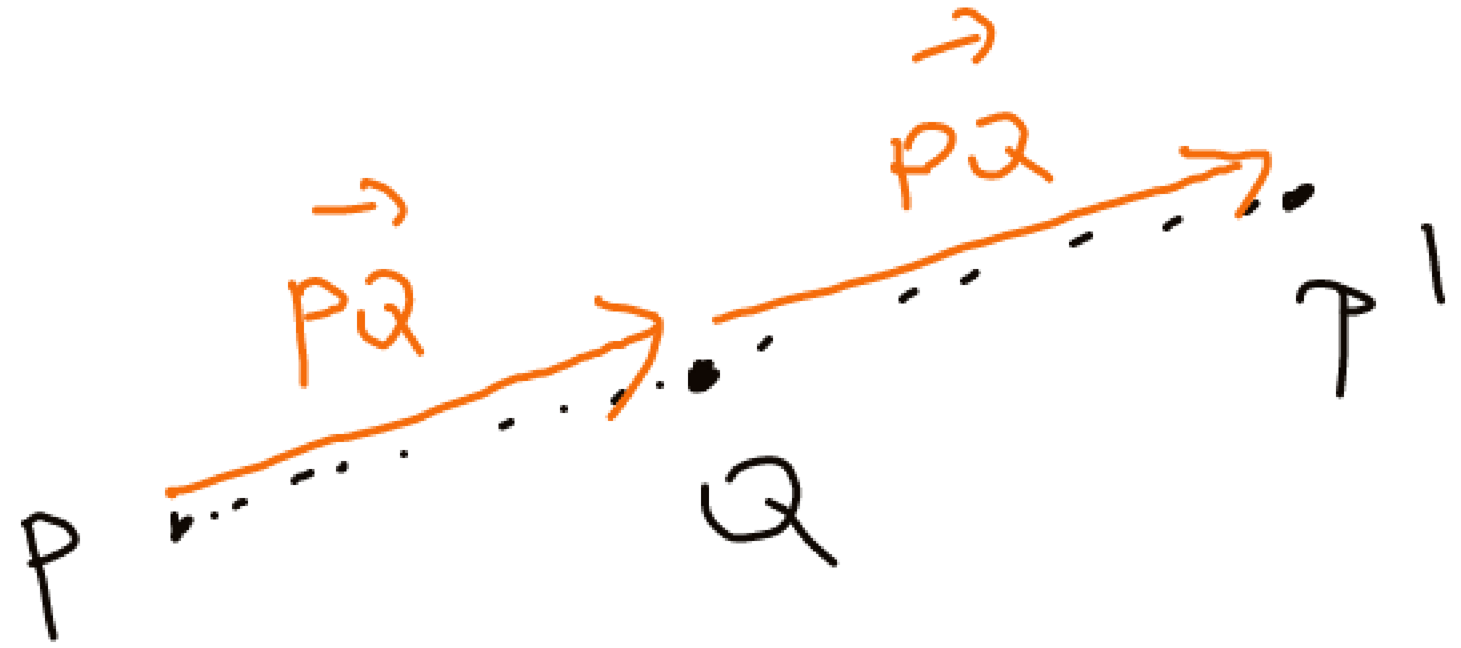
2° Jeśli $R_1 \neq R_2$, to rzutem ℓ na π jest prosta zawierająca R_1 i R_2

$$\ell: (x, y, z) = R_1 + t \cdot \overrightarrow{R_1 R_2}$$

XVIII/1 $\quad P=(1,2,0), \quad Q=(2,2,1)$

$\overrightarrow{PQ} = (1, 0, 1)$

$P' = Q + \overrightarrow{PQ} = (3, 2, 1)$



XVIII/2 $\quad P=(1,2,0)$

Q – nož. pravokotnice P na ℓ

$$\ell: \frac{x-2}{2}=\frac{y}{2}=\frac{z-2}{-1}$$

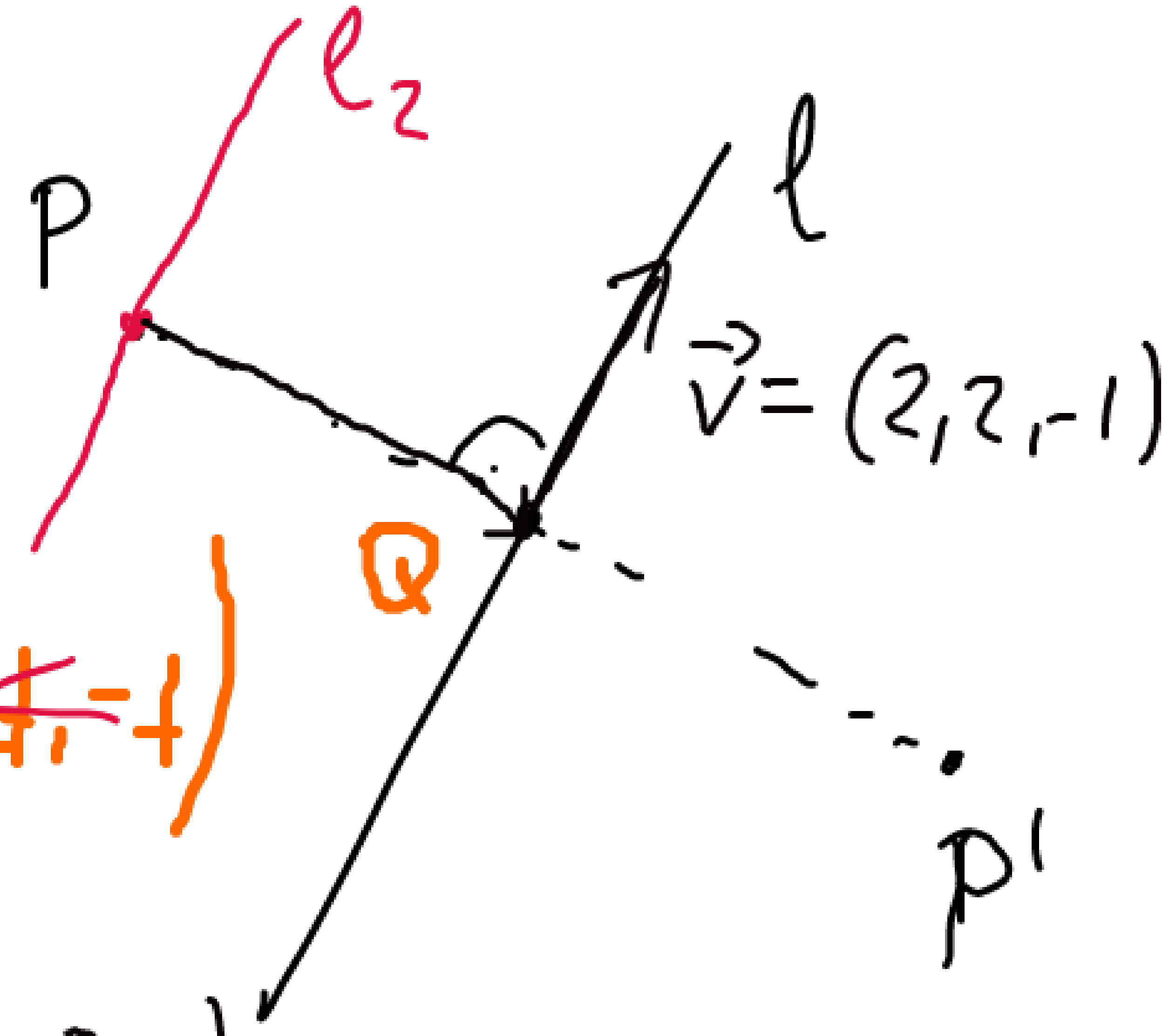
$\cancel{x = 1+2t}$

$\cancel{y = 2}$

$\ell_2 \begin{cases} \cancel{x = 1+2t} \\ \cancel{y = 2+2t} \\ \cancel{z = -t} \end{cases}$

$\cancel{Q(1+2t,\ 2+2t,\ -t)}$

$Q(2+2t,\ 2t,\ 2-t)$



$$\overline{PQ}=\frac{1}{2}\,\overline{PP'}$$

$$\overline{PQ}=(1+2t,\ 2t-2,\ 2-t) \ \wedge\ \overline{PQ}\circ(2,2,-1)=0 \Rightarrow t=\dots$$

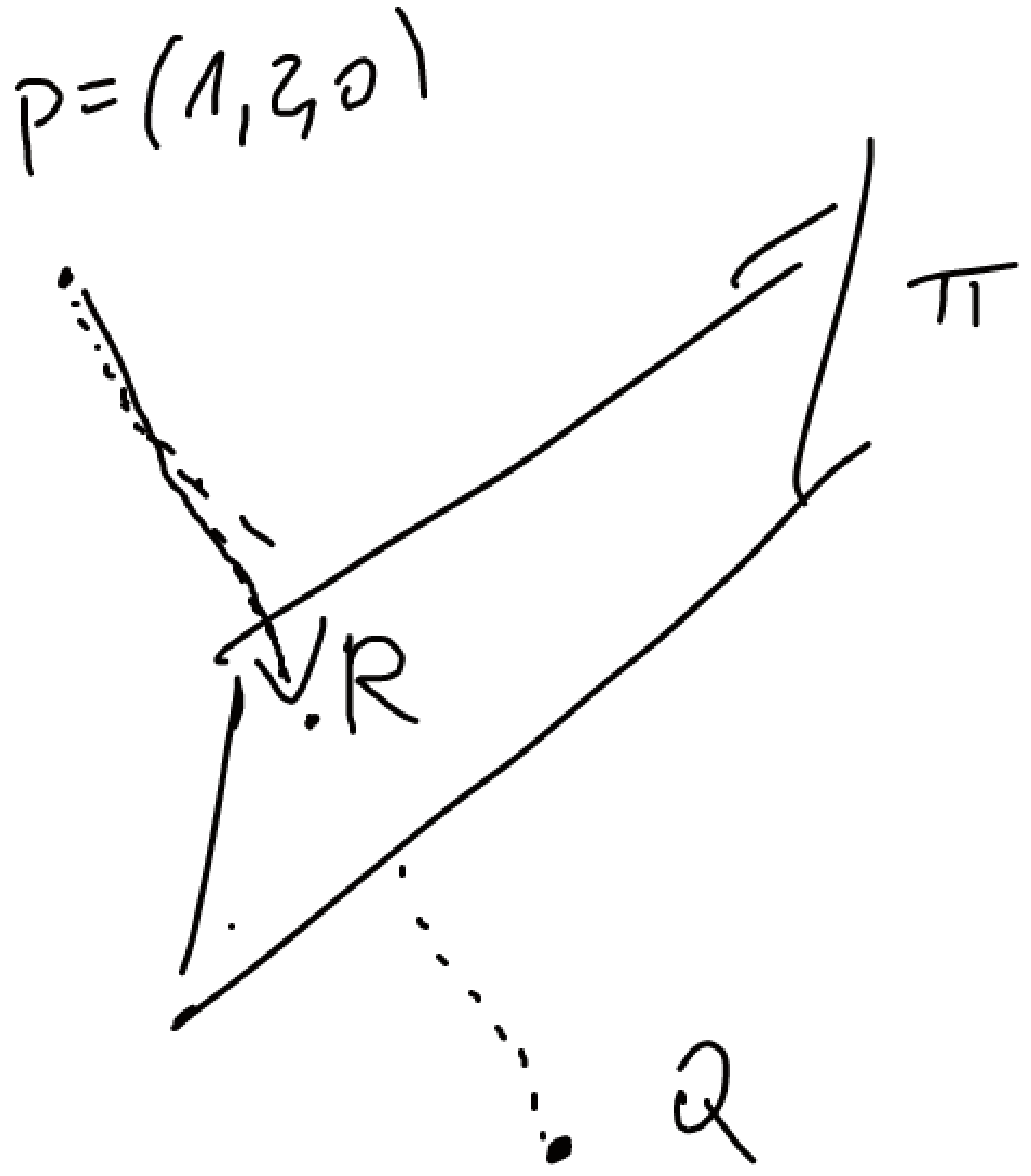
$\Rightarrow \overline{PQ}=(\dots)$

$$P'=Q+\overline{PQ}$$

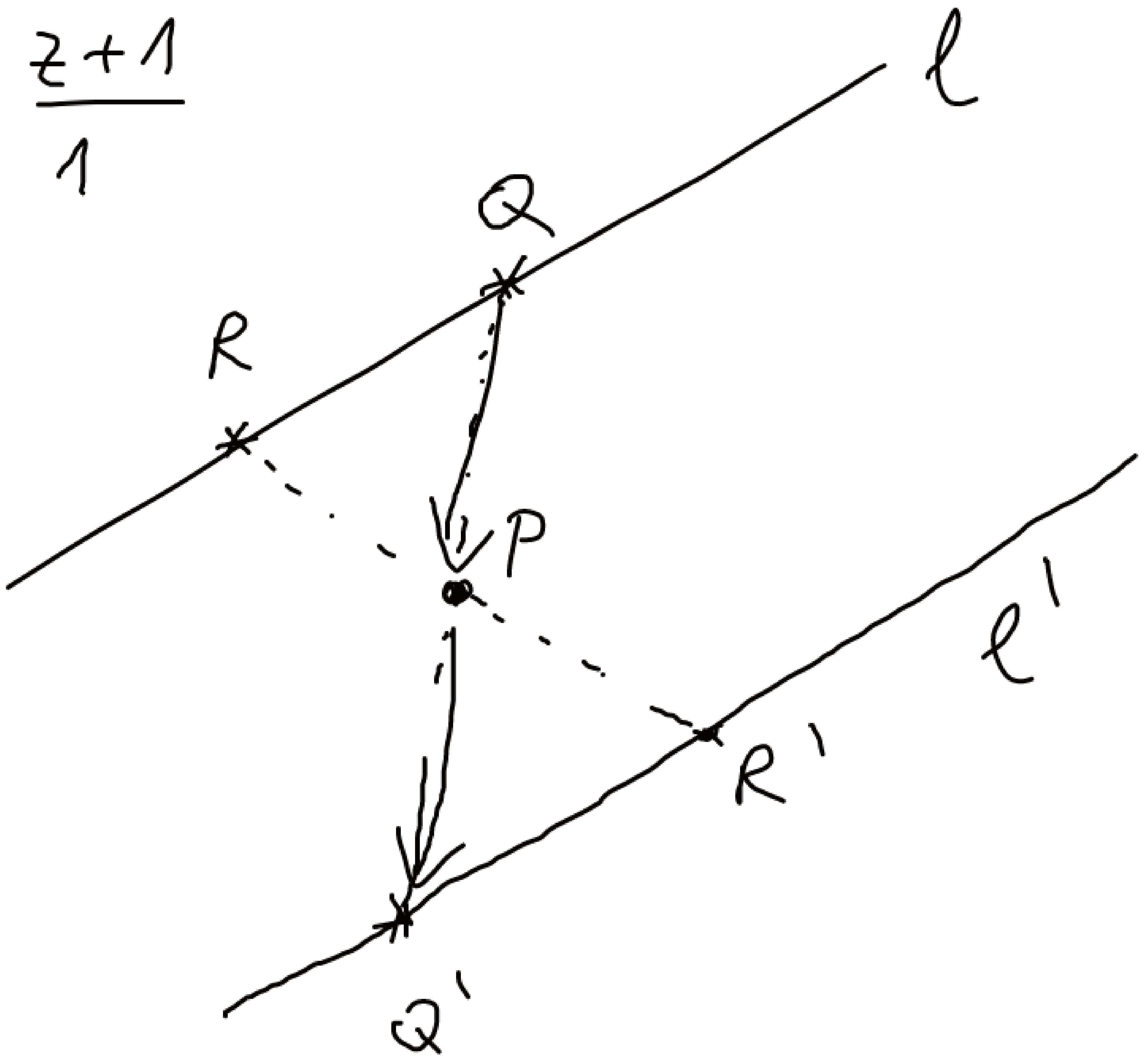
XVIII/3

Znajdujemy rzut prostokątny R punktu P na π (jak w XVII/1)

$$Q = R + \overrightarrow{PR} = \ldots$$



XIX $P = (1,1,0)$ $\quad \ell: \dfrac{x-1}{1} = \dfrac{y-1}{2} = \dfrac{z+1}{1}$



$R = (1,1,-1)$

$$\begin{cases} \dfrac{x-1}{1} = \dfrac{y-1}{2} = \dfrac{z+1}{1} = 1 \\ x = 2 \\ y = 3 \\ z = 0 \end{cases}$$

$Q = (2,3,0)$

$R' = P + \overrightarrow{RP} = \ldots$

$Q' = P + \overrightarrow{QP} = \ldots$

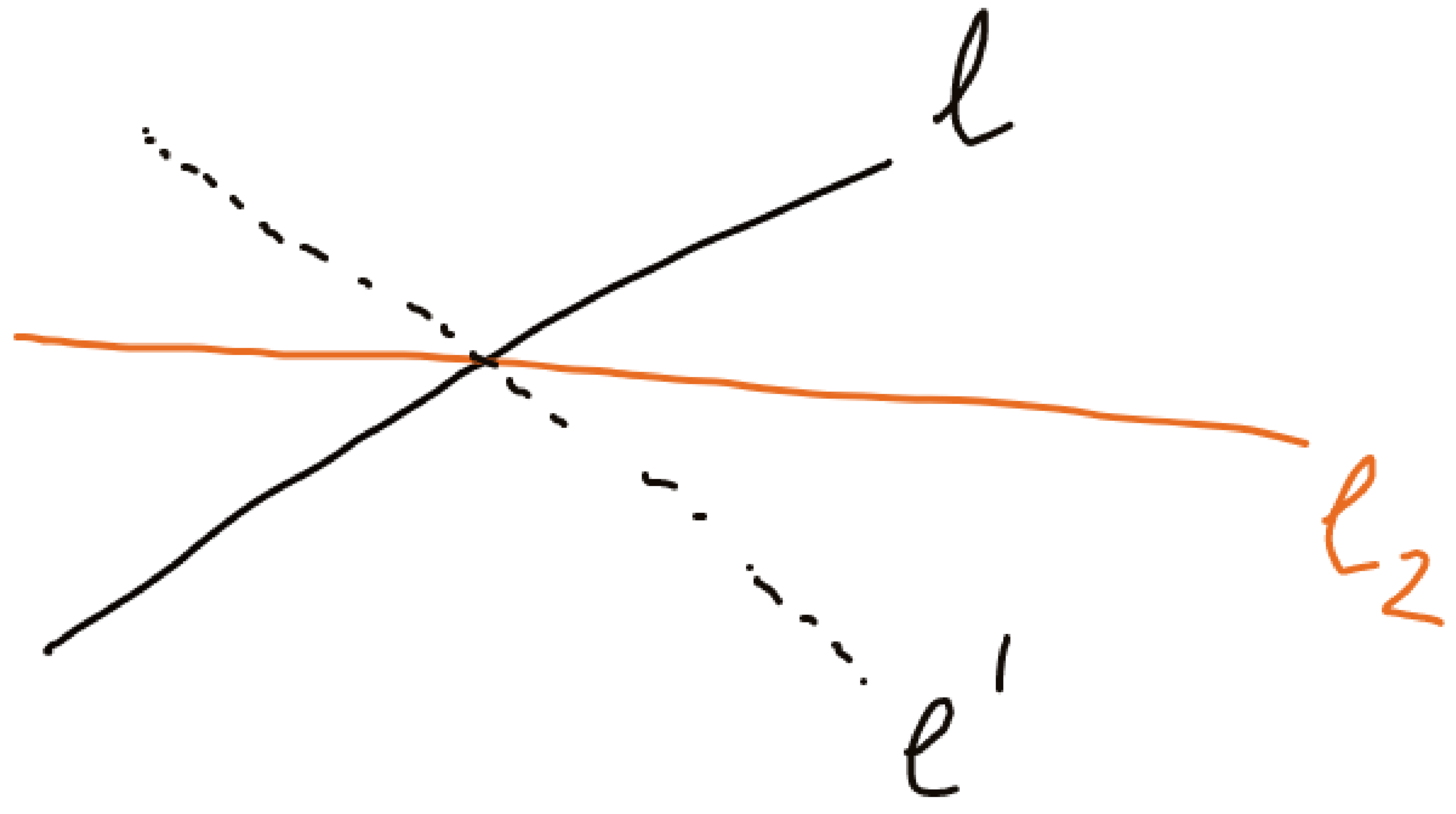
$\ell': (x,y,z) = R' + t \cdot (\overrightarrow{R'Q'})$

XIX-2)

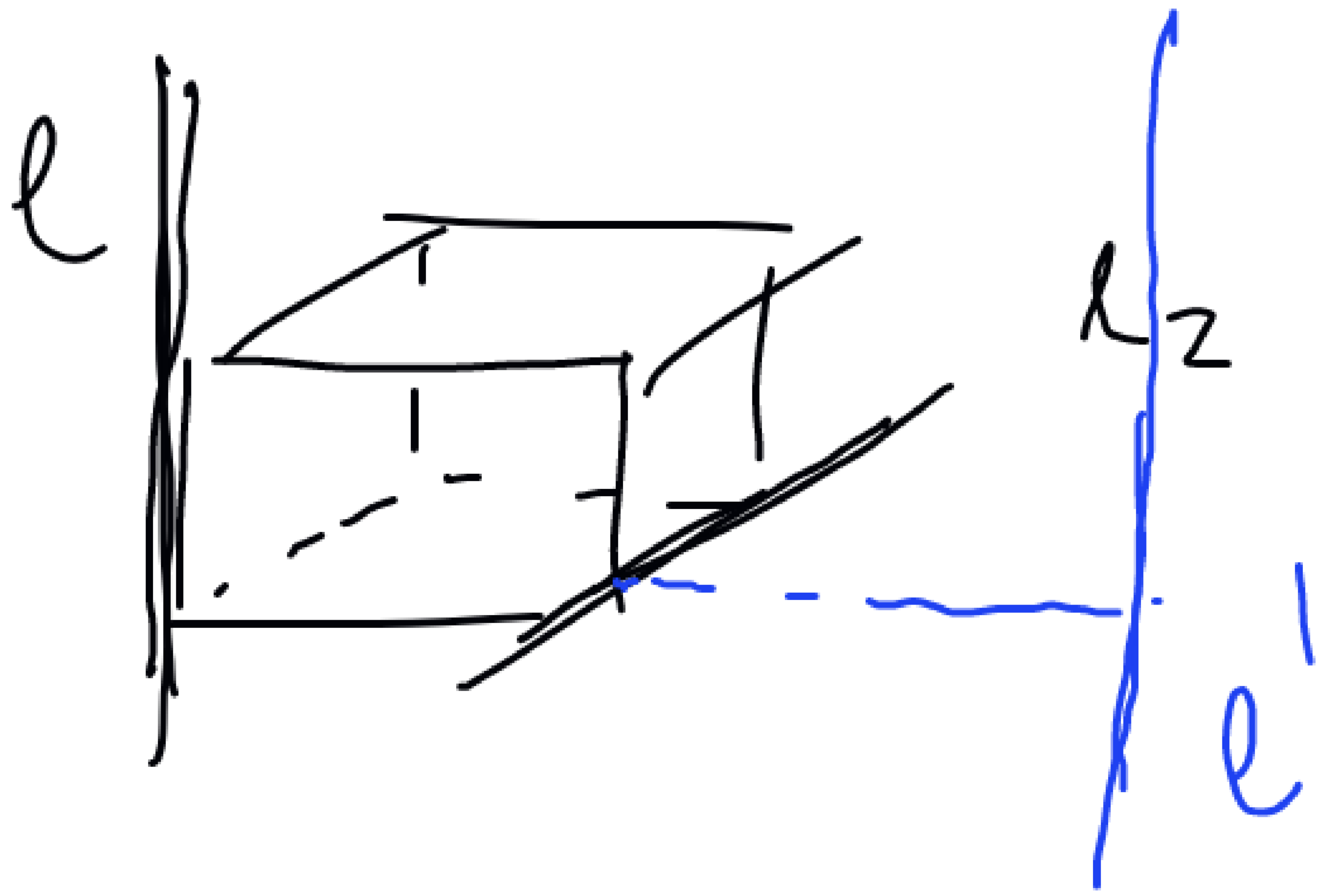
Wybieramy dwa różne punkty $P, Q \in l$

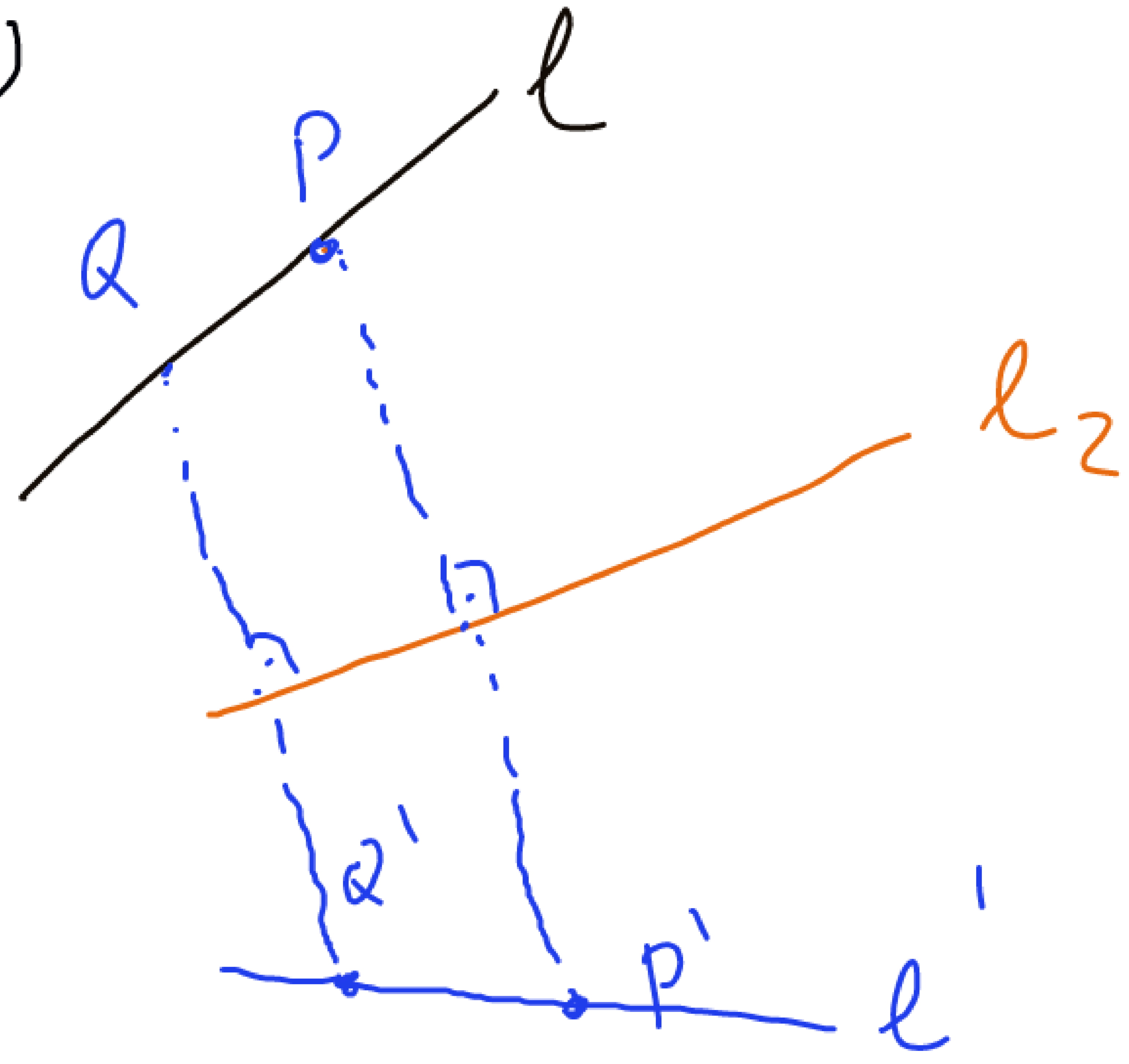
i odbijamy je symetrycznie wzgl.

prostej $l_2: \frac{x}{2} = \frac{y-2}{1} = \frac{z+1}{1}$



dostajemy dwa punkty P', Q' z szukanej prostej



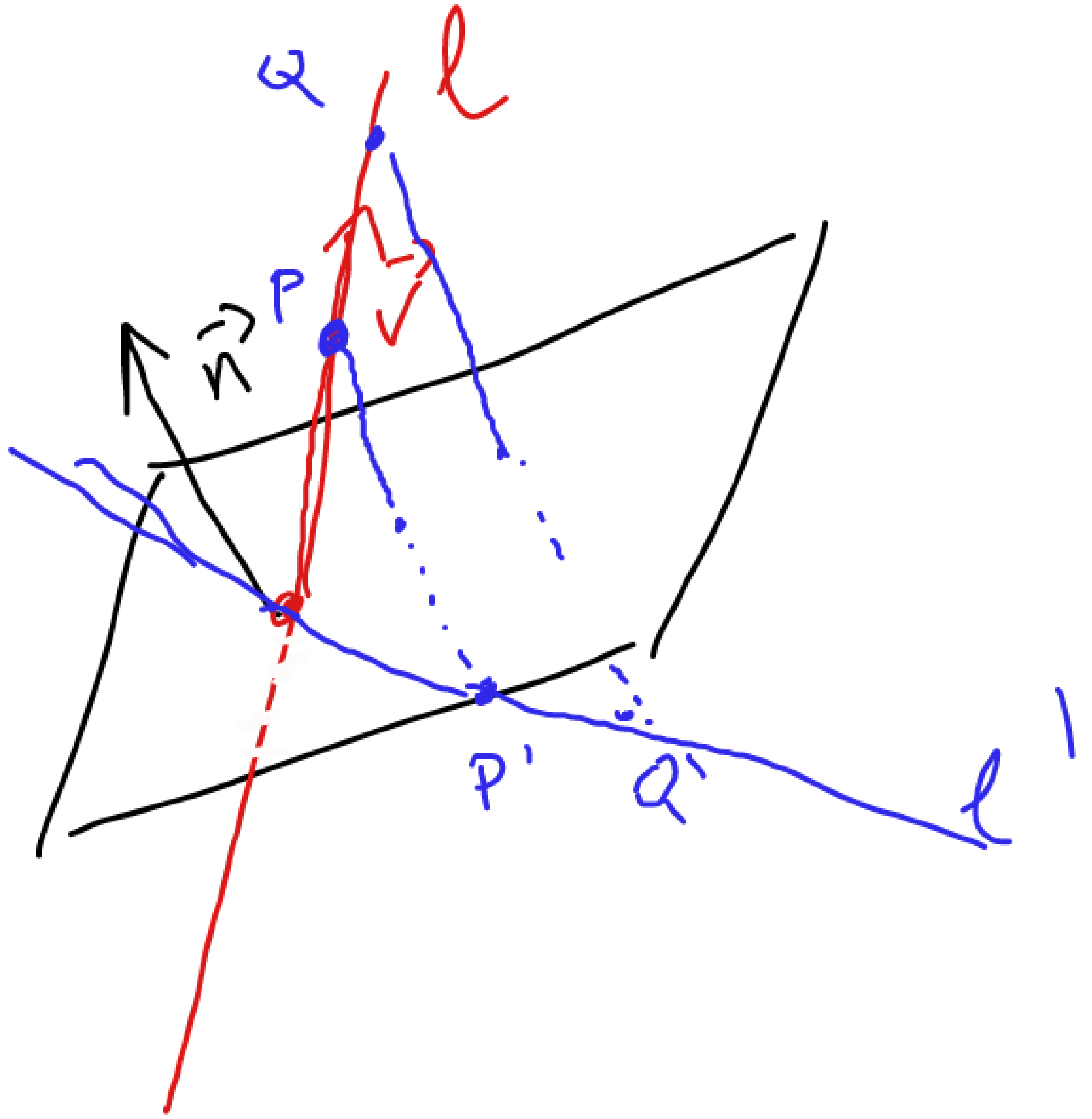


XIX | 3

$$l: \frac{x-1}{1} = \frac{y-1}{2} = \frac{z+1}{1} \qquad \vec{v} = (1,2,1)$$

$$\pi: x-2z+1 \qquad \vec{n} = (1,0,-2)$$

$$\vec{v} \circ \vec{n} = 1-2 = -1 \neq 0$$



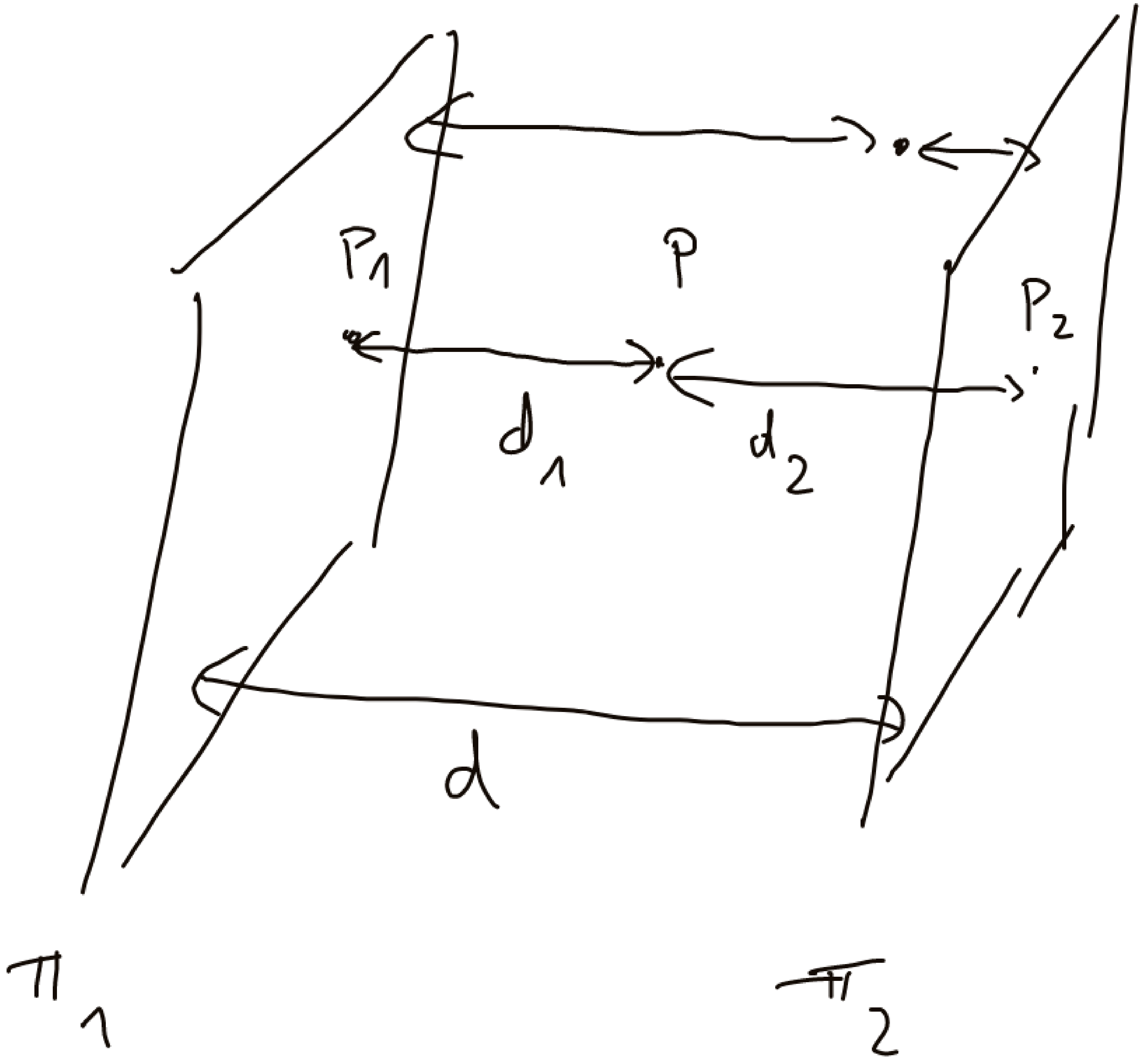
P, Q - dwa róne punkty z l

P', Q' - obrazy symetryczne P, Q wzgl. π

$$l': (x,y,z) = P' + t \cdot \overrightarrow{P'Q'}$$

XX

P



P

$d_1+d_2=d \iff$ P leży pomiędzy π_1 i π_2 (lub na π_1, π_2)

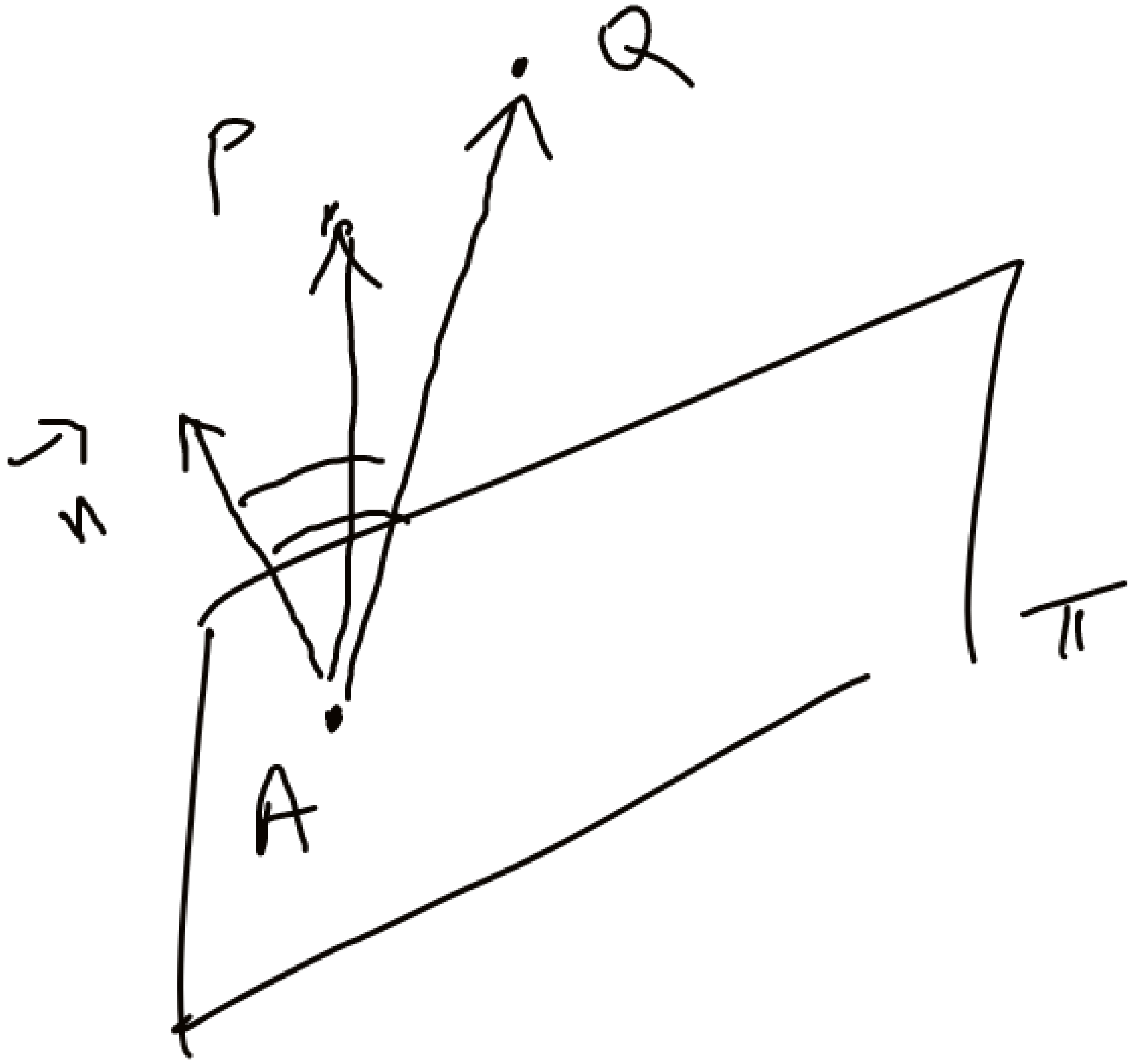
$d_1+d_2>d \iff$ P nie leży pomiędzy | $d_1>d_2$

XXI

$\vec{AP}\circ\vec{n}$

$\vec{AQ}\circ\vec{n}$

jeśli mają taki sam znak, to P, Q leżą po tej samej stronie π



$$2x+3z-7=0$$

$$d\left((-2,4,3),\pi\right) = \frac{\left|2\cdot(-2)+0\cdot 4+3\cdot 3-7\right|}{\sqrt{2^2+0^2+3^2}}$$