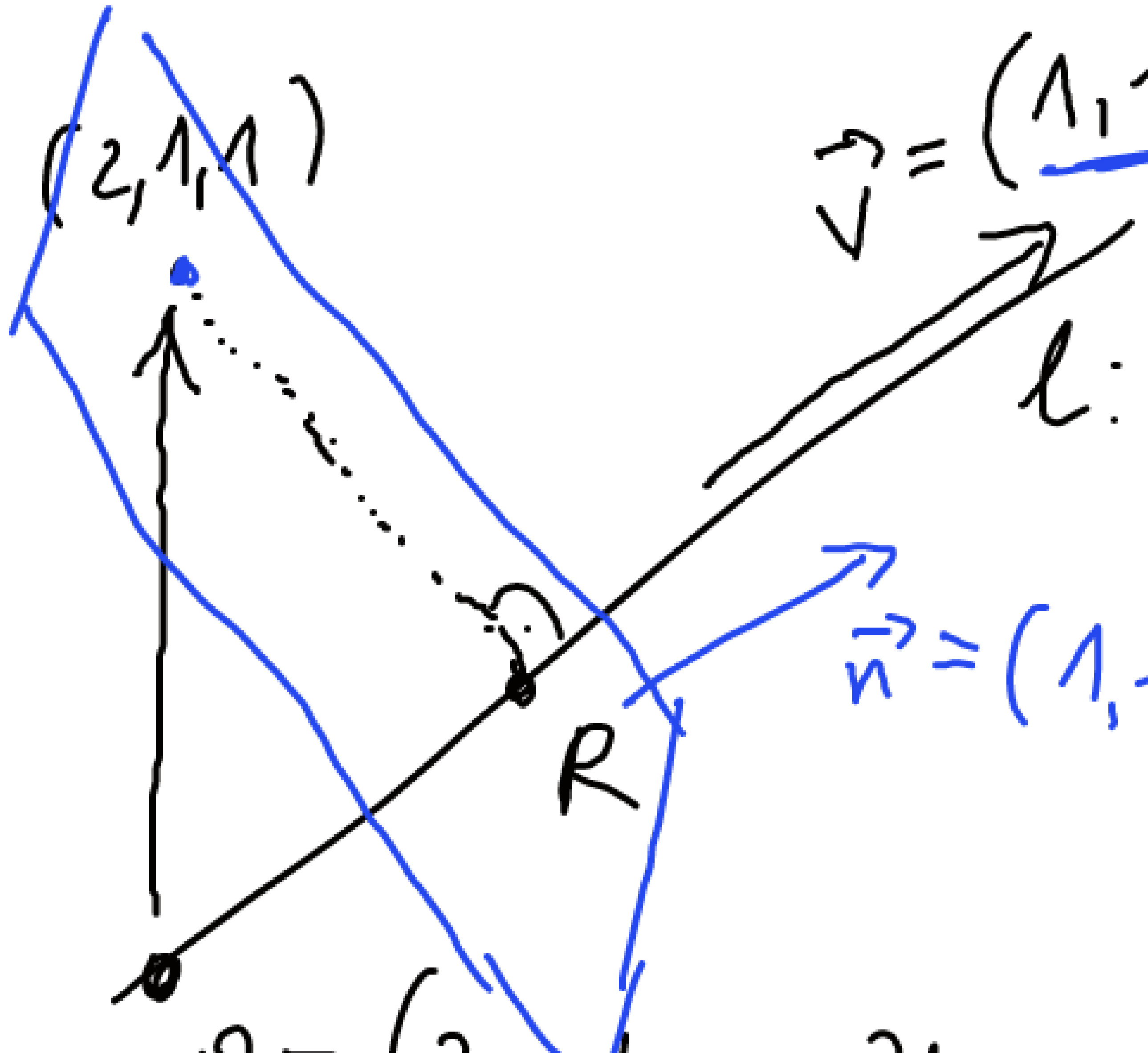
XVII|2

$\vec{QP} \perp \vec{v}$, tzn. $\vec{QP} \circ \vec{v} = 0$

$P = (2,1,1)$

$\vec{v} = (1,-2,1)$



$l: \frac{x-2}{1} = \frac{y}{-2} = \frac{z+1}{1} = t$

$\vec{n} = (1,-2,1)$

$Q = (2+t, -2t, -1+t)$, $t \in \mathbb{R}$

$\vec{QP} = P - Q = (-t, 1+2t, 2-t)$

$0 = \vec{QP} \circ \vec{v} = -t - 2(\cancel{1} + 2t) + (\cancel{2} - t)$

$0 = -6t$

$t = 0$

$R = (2, 0, -1)$

II sposób

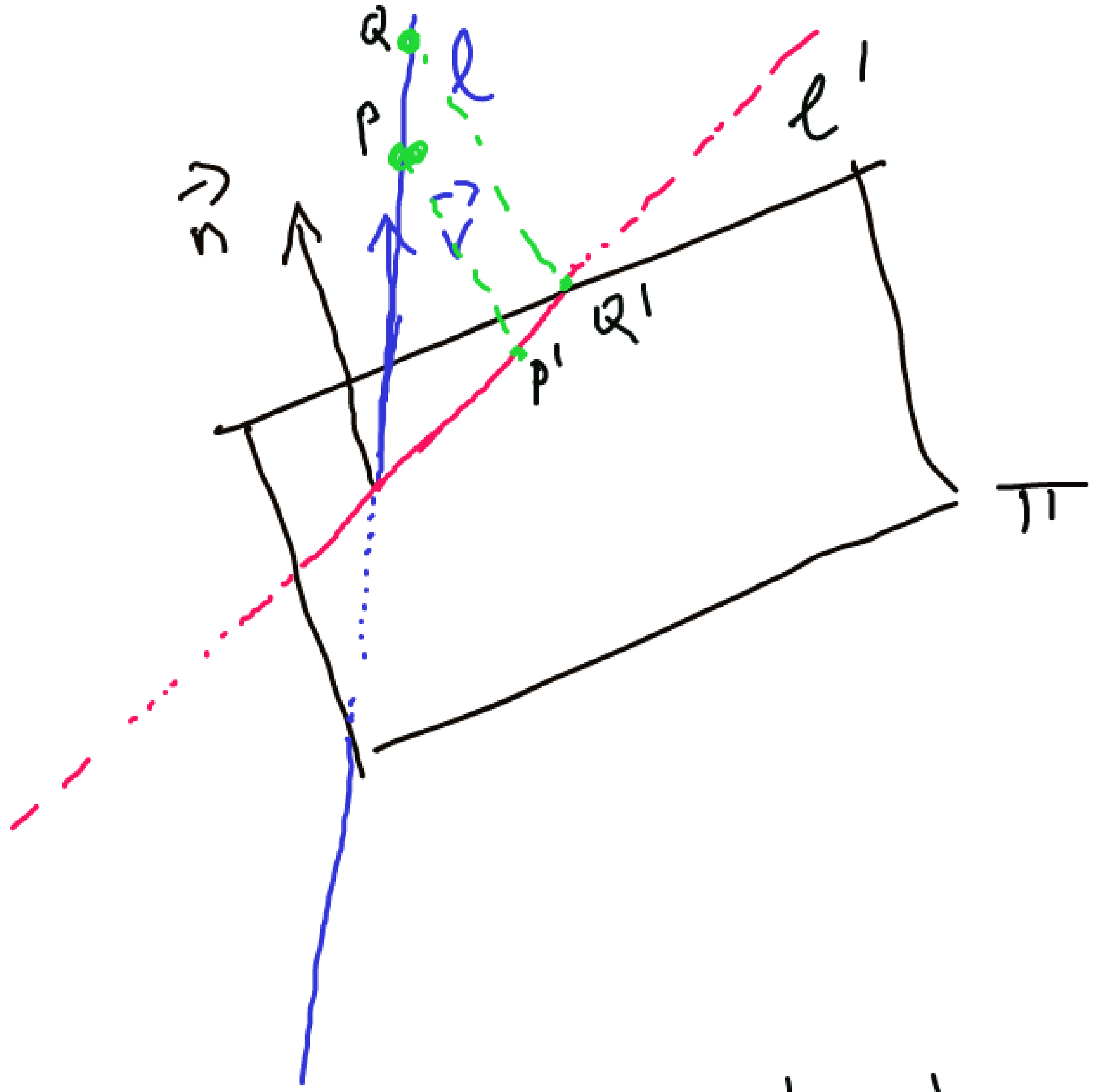
$\pi: (x-2) - 2(y-1) + (z-1) = 0$

$\pi \cap l = \{R\}$

XVII/3

$$\frac{x+2}{1} = \frac{y-1}{-2} = \frac{z}{1} \qquad \vec{v} = (1,-2,1)$$

$$\pi: \ 2x - z + 3 = 0 \qquad \vec{n} = (2,0,-1)$$



$$\vec{n} \circ \vec{v} = 2 - 1 = 1 \neq 0$$

Bierzemy dwa różne punkty $P, Q \in \ell$ i rzutujemy je na płaszczyznę, otrzymamy punkty P', Q'

Jeśli $P' = Q'$, to wtedy jest pojedynczy punkt P'

Jeśli $P' \neq Q'$, to $\ell: (x,y,z) = P' + t\,\overrightarrow{P'Q'}$

Jak znaleźć P, Q?

$$1 = \frac{x+2}{1} = \frac{y-1}{-2} = \frac{z}{1}$$

$\downarrow$

$x = -1$

$y = -1$

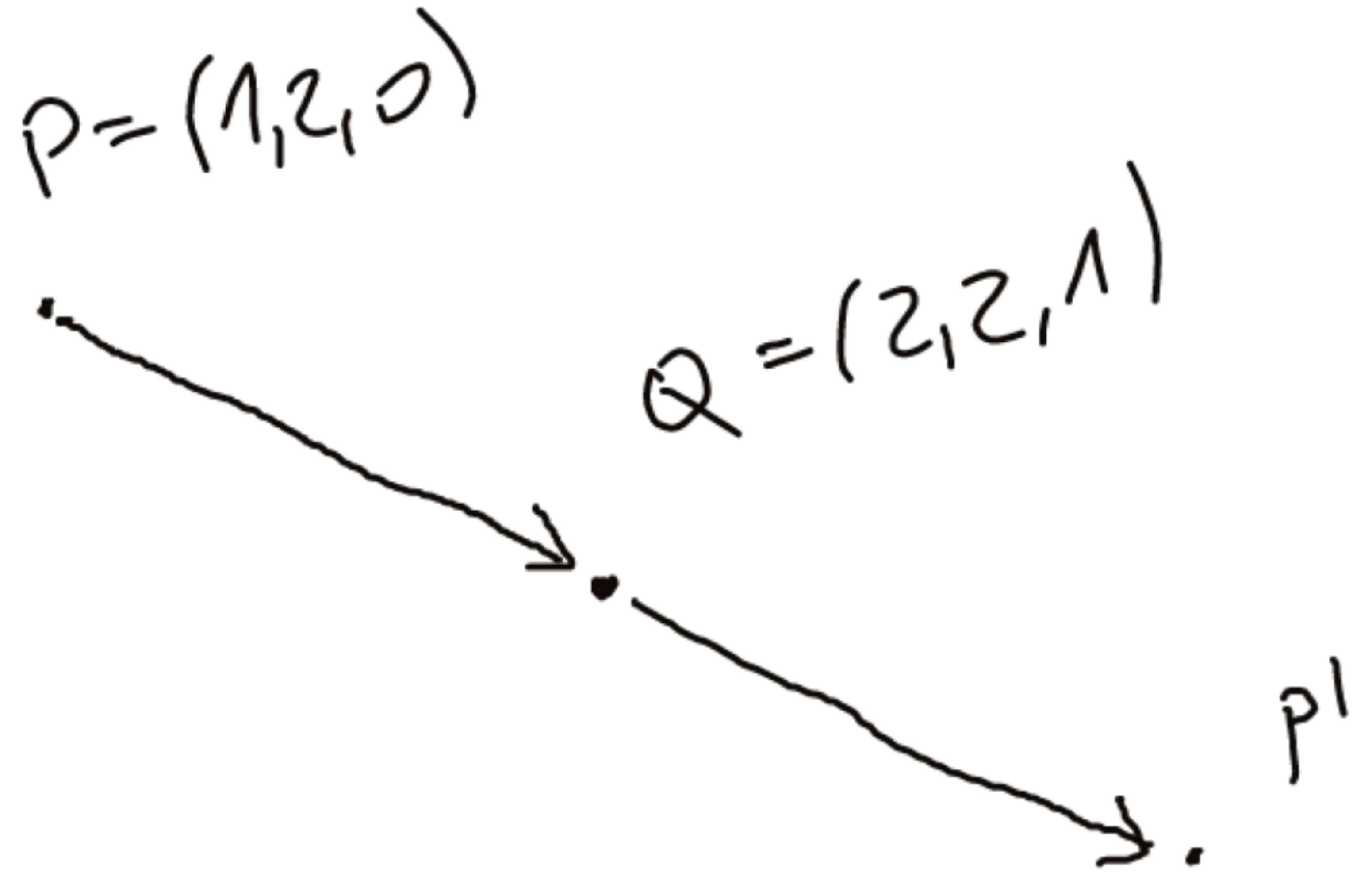
$z = 1$

$P = (-2,1,0)$

$Q = (-1,-1,1)$

XVIII - 1

$$P' = Q + \overrightarrow{PQ}$$



$$\overrightarrow{PQ} = [1,0,1]$$

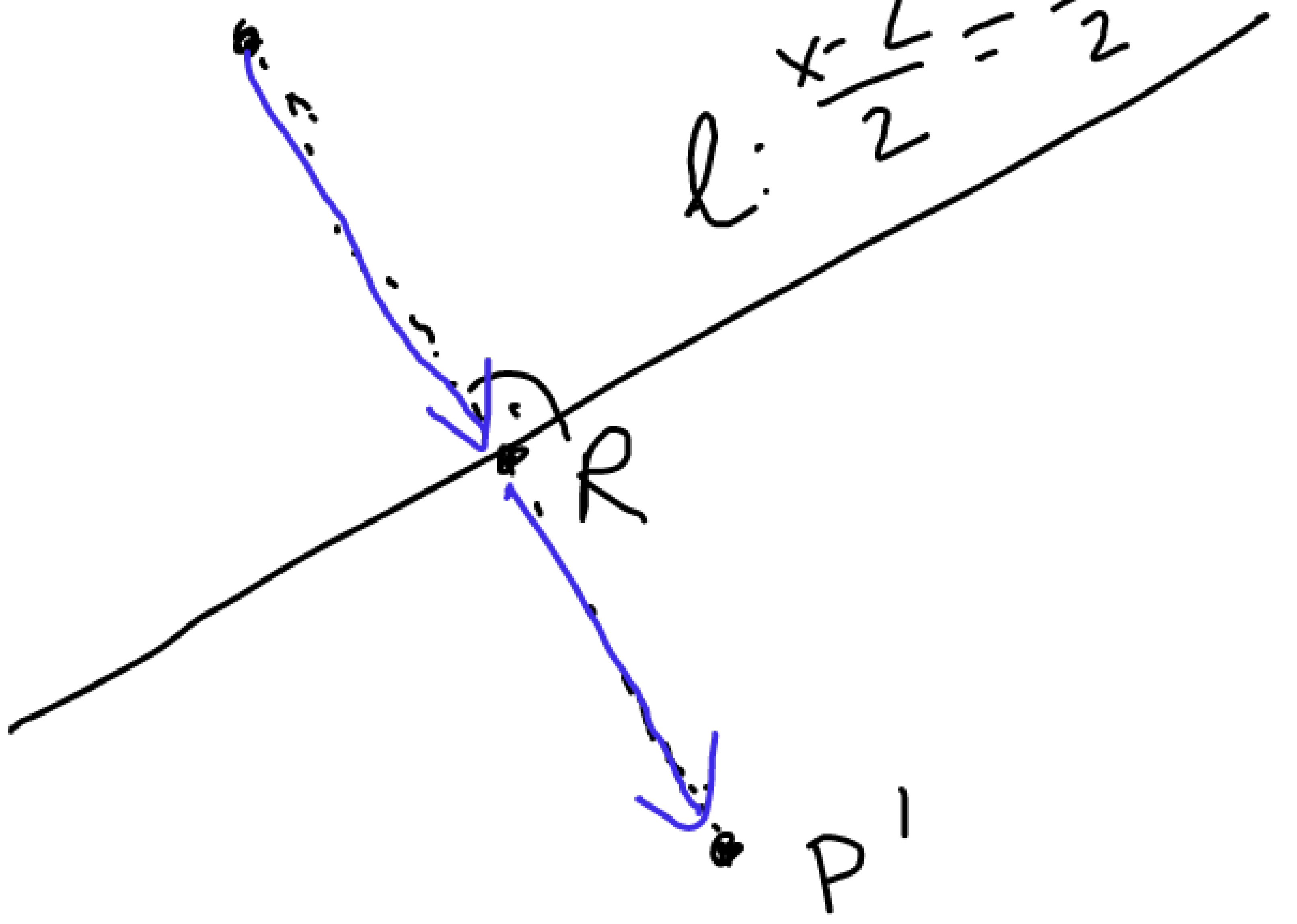
$$P' = (2,2,1] + (1,0,1) = [3,2,2]$$

XVIII/2

Znajdujemy rzut prostokątny R punktu P na ℓ.

$P = (1,2,0)$

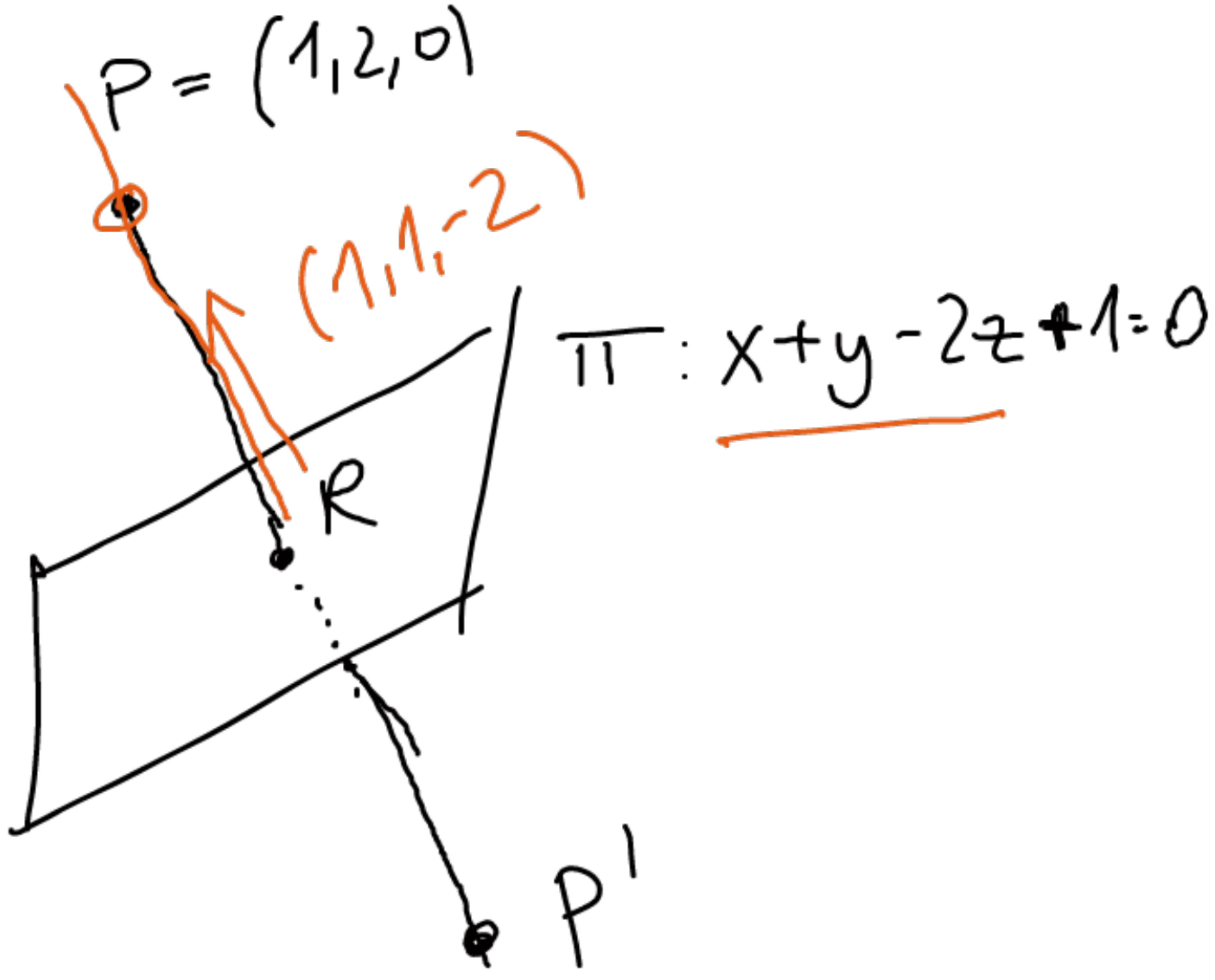
$\ell: \frac{x-2}{2} = \frac{y}{2} = \frac{z-2}{-1}$



$P' = R + \overrightarrow{PR}$

XVIII-3

Znajdujemy rzut prostokątny R ~~pł~~ punktu P na pł: π



$$\ell: (x,y,z) = (1,2,0) + t(1,1,-2)$$

$$\ell \cap \pi = \{R\}$$

Dalej

$$P' = R + \overrightarrow{PR}$$

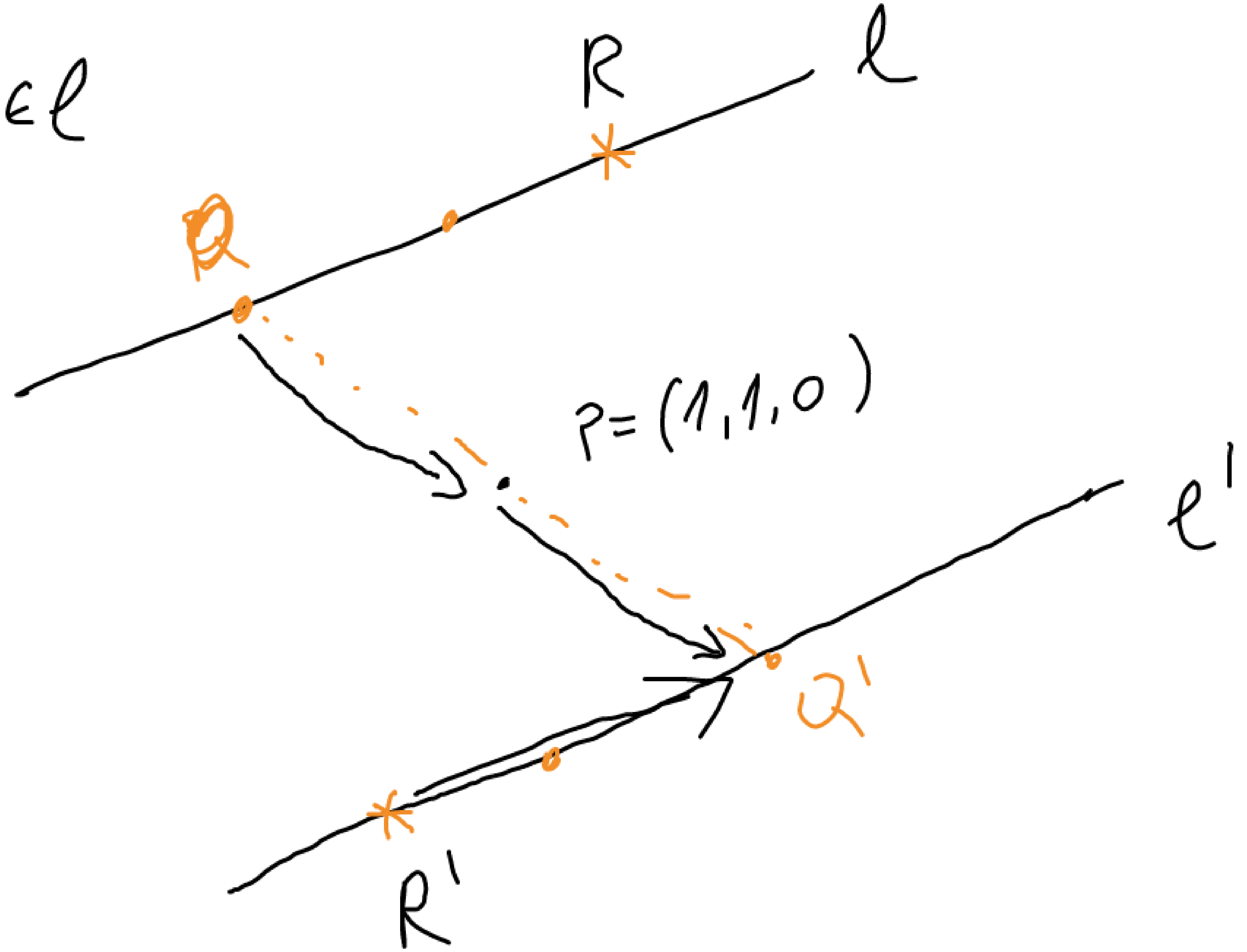
XIX/1

Znajdujemy dwa różne punkty $R, Q \in \ell$ i ich obrazy sym. wzgl. P:

$$Q' = P + \vec{QP}$$

$$R' = P + \vec{RP}$$

$$\ell' : (x,y,z) = R' + t\,\vec{R'Q'}$$

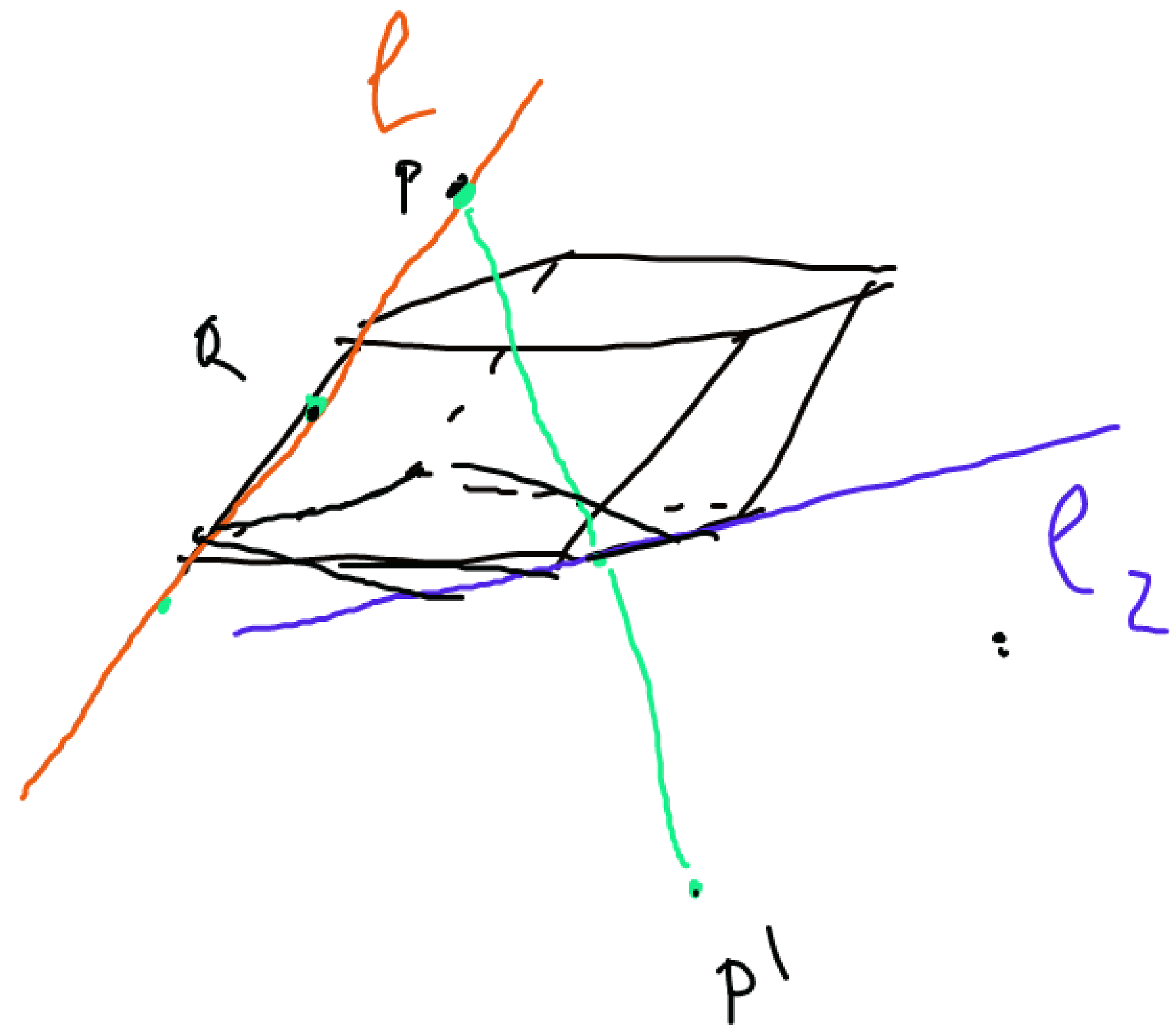


XIX/2

$\ell: \frac{x-1}{1} = \frac{y-1}{2} = \frac{z+1}{1} = t \quad (1,2,1)$

$\ell_2: \frac{x}{2} = \frac{y-2}{1} = \frac{z+1}{1} \quad (2,1,1)$

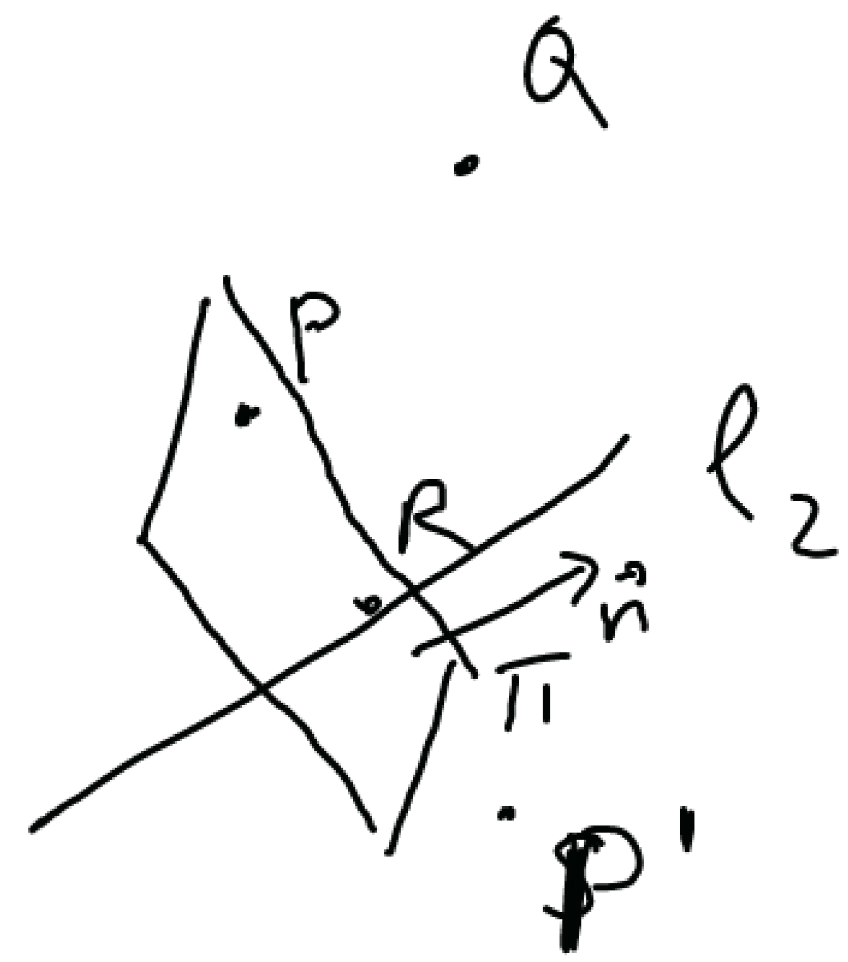
$\ell_2: \begin{cases} x = 2s \\ y = 2+s \\ z = -1+s \end{cases}$



Punkty z ℓ mają postać

$$\begin{cases} x = 1+t \\ y = 1+2t \\ z = -1+t \end{cases}, \quad t \in \mathbb{R}$$

Np. $P = (1,1,-1)$, $Q = (2,3,0) \in \ell$



Odbijamy je symetrycznie wzgl. ℓ_2 i dostajemy P', Q'

• Napiszmy r-nie płaszczyzny $\pi \ni P$, $\pi \perp \ell_2$

$\vec{n} = (2,1,1)$

$\pi: 2(x-1) + (y-1) + (z+1) = 0$

$\pi \cap \ell_2 = \{R\}$ $\quad 2(2s-1) + (2+s-1) + (-1+s+1) = 0$

$6s - 1 = 0 \quad s = 1/6 \quad R = \left(\frac{2}{6}, \frac{13}{6}, -\frac{5}{6}\right)$

$\vec{PR} = \left(-\frac{4}{6}, \frac{7}{6}, \frac{1}{6}\right)$

$P' = R + \vec{PR} = \left(\frac{2}{6}, \frac{13}{6}, -\frac{5}{6}\right) + \left(-\frac{4}{6}, \frac{7}{6}, \frac{1}{6}\right) = \left(-\frac{2}{6}, \frac{20}{6}, -\frac{4}{6}\right)$

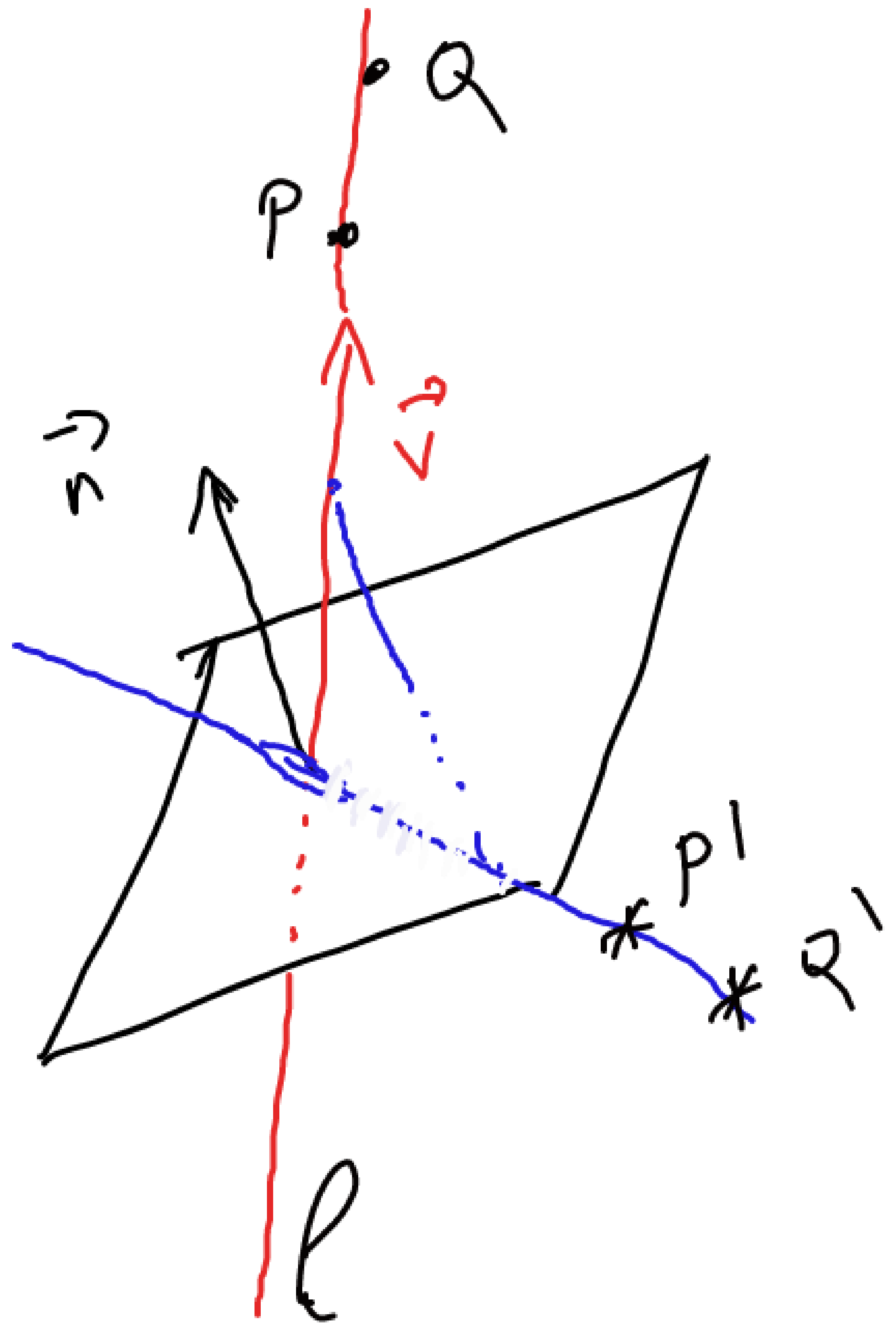
$= \left(-\frac{1}{3}, \frac{10}{3}, -\frac{2}{3}\right)$

XIX13

$$l: \frac{x-1}{1} = \frac{y-1}{2} = \frac{z+1}{1} \qquad \vec{v} = (1,2,1)$$

$$\pi: x - 2z + 1 = 0 \qquad \vec{n} = (1, 0, -2)$$

$$\vec{v} \circ \vec{n} = 1 - 2 = -1 \neq 0$$

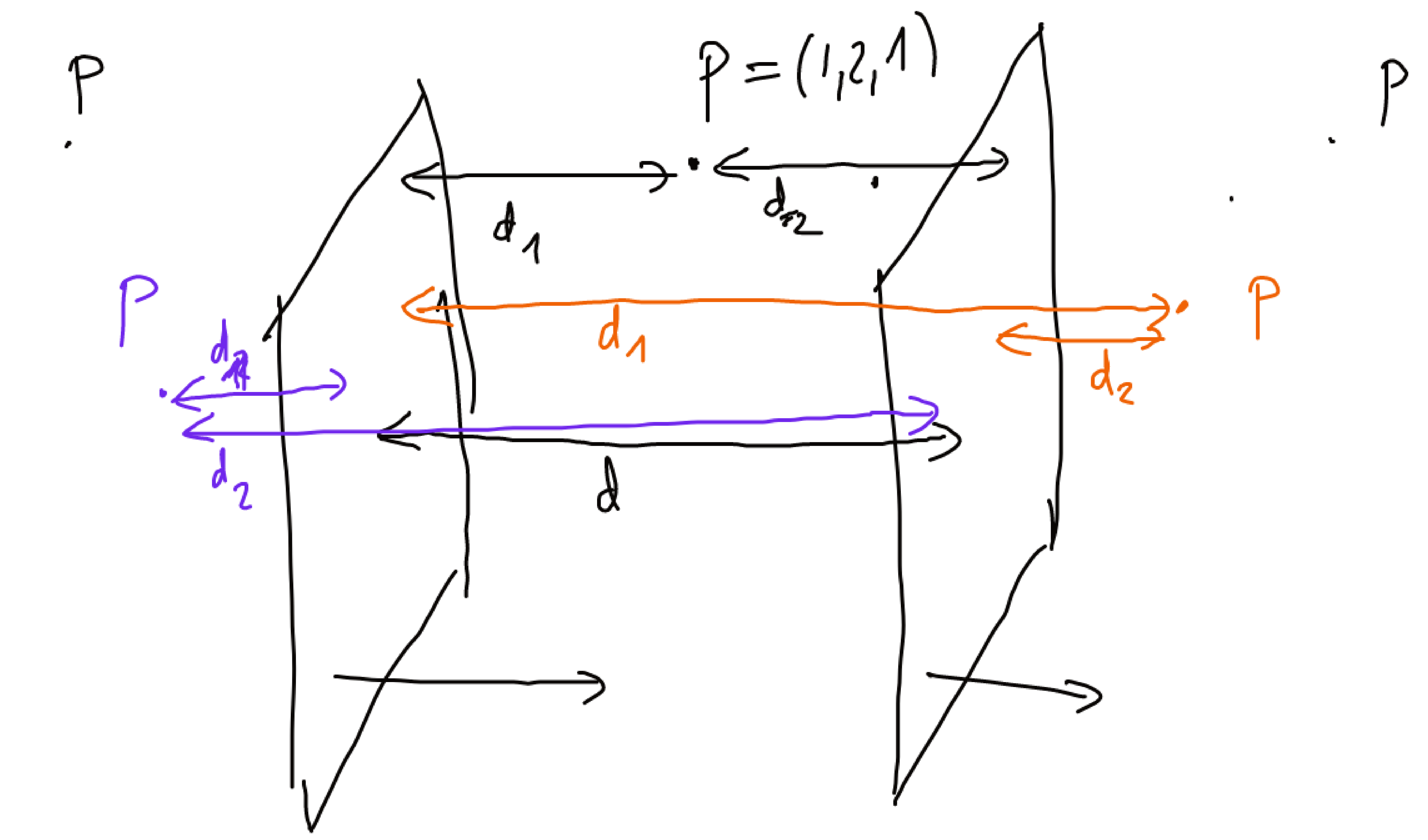


$P, Q \in l$, $P \neq Q$

szukamy P', Q' – odbicia sym. P, Q względem π

$$l: (x,y,z) = P' + t \cdot \overrightarrow{P'Q'}$$

XX



$\pi_1: 6x-3y+6z+3=0$ $\pi_2: 4x-2y+4z-2=0$

$\vec{n}_1=(6,-3,6)$ $\vec{n}_2=(4,-2,4)=\frac{2}{3}\cdot\vec{n}_1$

$d=d_1+d_2 \rightarrow$ P leży między π_1 i π_2 (lub na π_1, π_2)

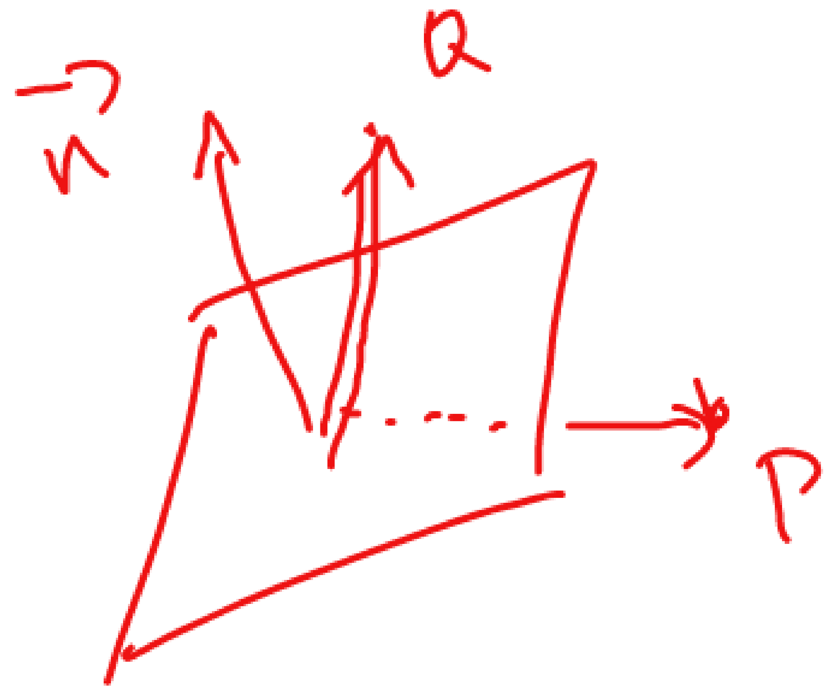
$d<d_1+d_2 \rightarrow$ P nie leży -||- ————

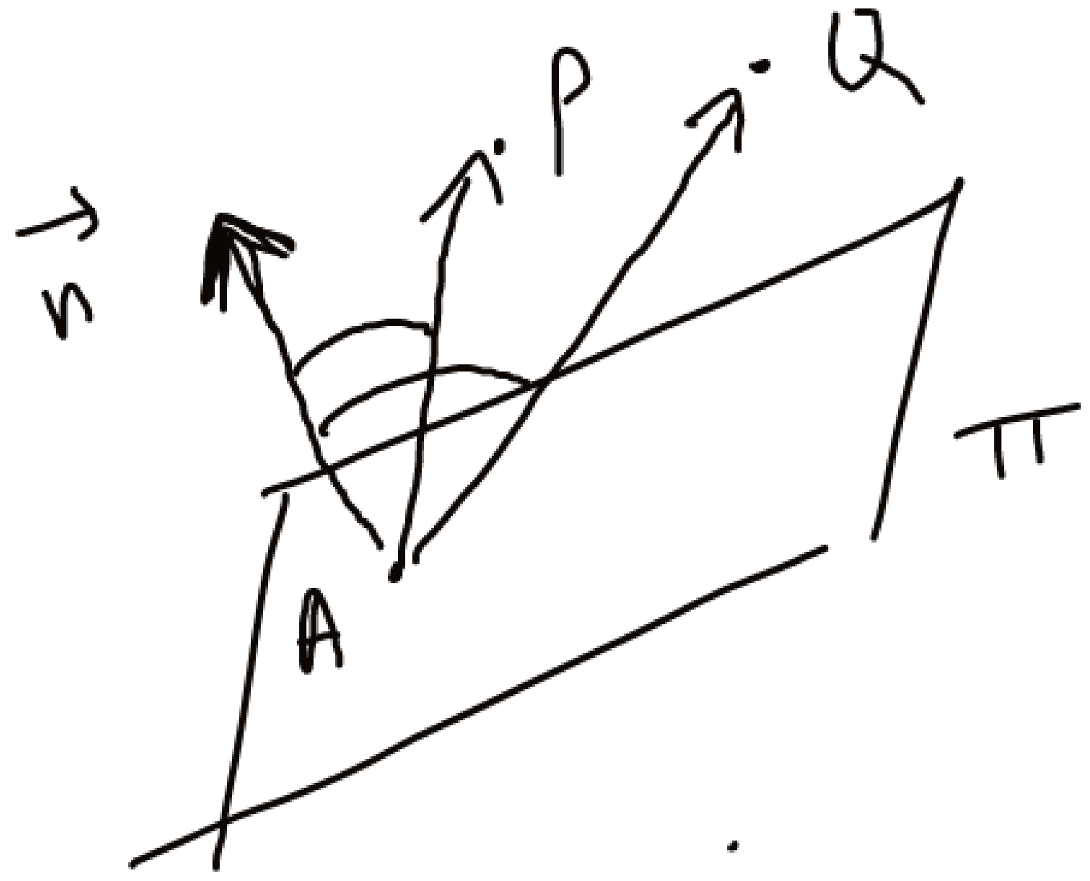
XXI

$P=(-2,4,3)$

$Q=(1,-2,2)$

$\pi:\ \boxed{2x+3z-7}=0$





$\vec{AP}=(-4,4,2)$

$\vec{AP}\circ\vec{n}=-8+0+6=-2<0$

$\Rightarrow \sphericalangle(\vec{AP},\vec{n})$ jest rozwarty

np. $A=(2,0,1)\in\pi$

$\vec{n}=(2,0,3)$

$\vec{AQ}=(-1,-2,1)$

$3 \quad 1>0$

$\vec{AQ}\circ\vec{n}=-2+0+\cancel{1}=\cancel{-1}<0$

$\Rightarrow \sphericalangle(\vec{AQ},\vec{n})$ jest ~~też rozwarty~~ ostry

$\Rightarrow$ P, Q nie leżą po jednej stronie π

$\underbrace{2x+3z-7}_{f(x,y,z)}=0$

$(2,0,3)$

$2x+3z-7>0$

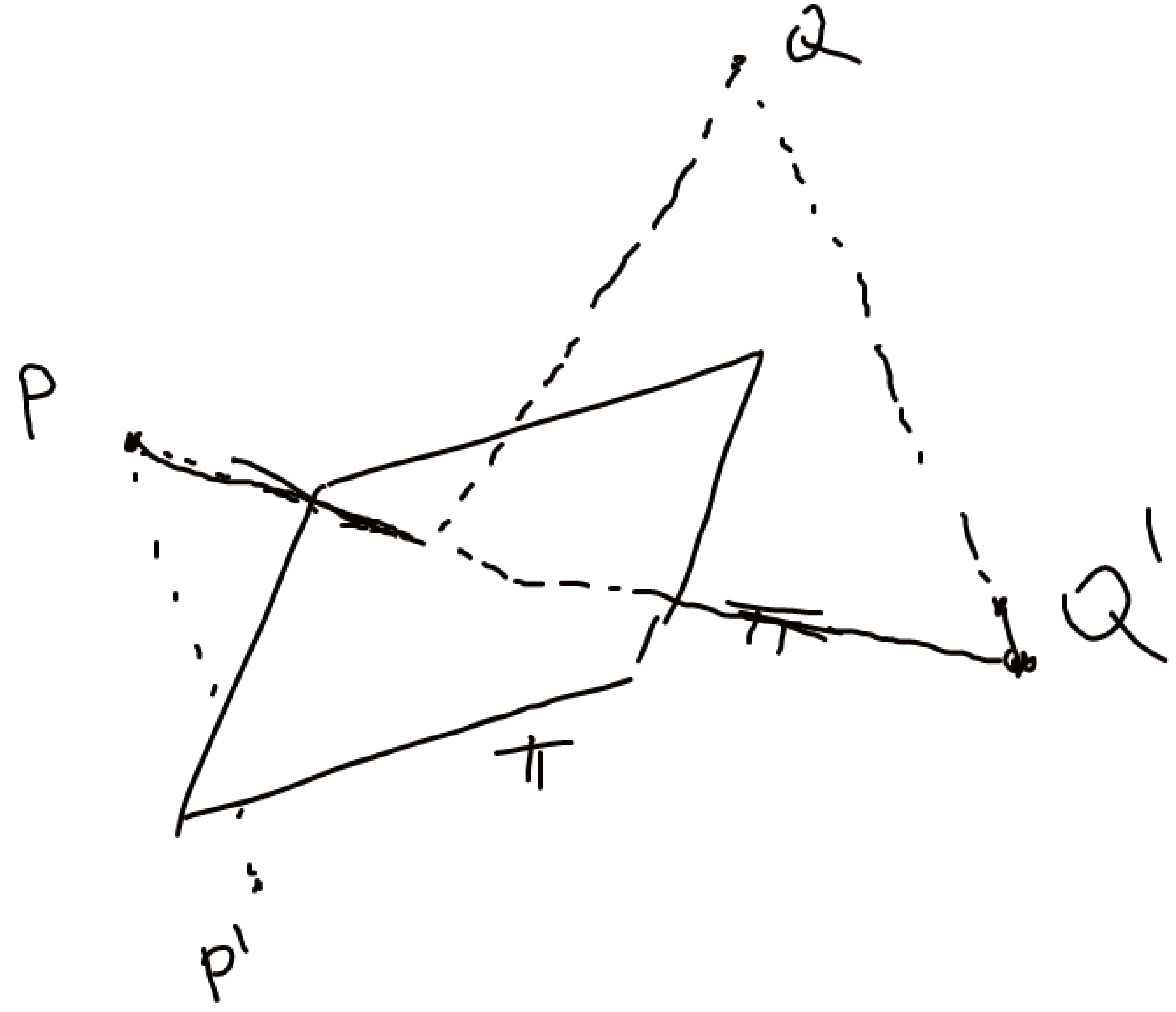
$f(x,y,z)=0$

$2x+3z-7<0$

XXIV

Q' – obraz Q przez symetrię względ. π

$l = (x,y,z) = P + t\cdot\vec{PQ}$ $\longrightarrow$ prosta zawierająca promień padający



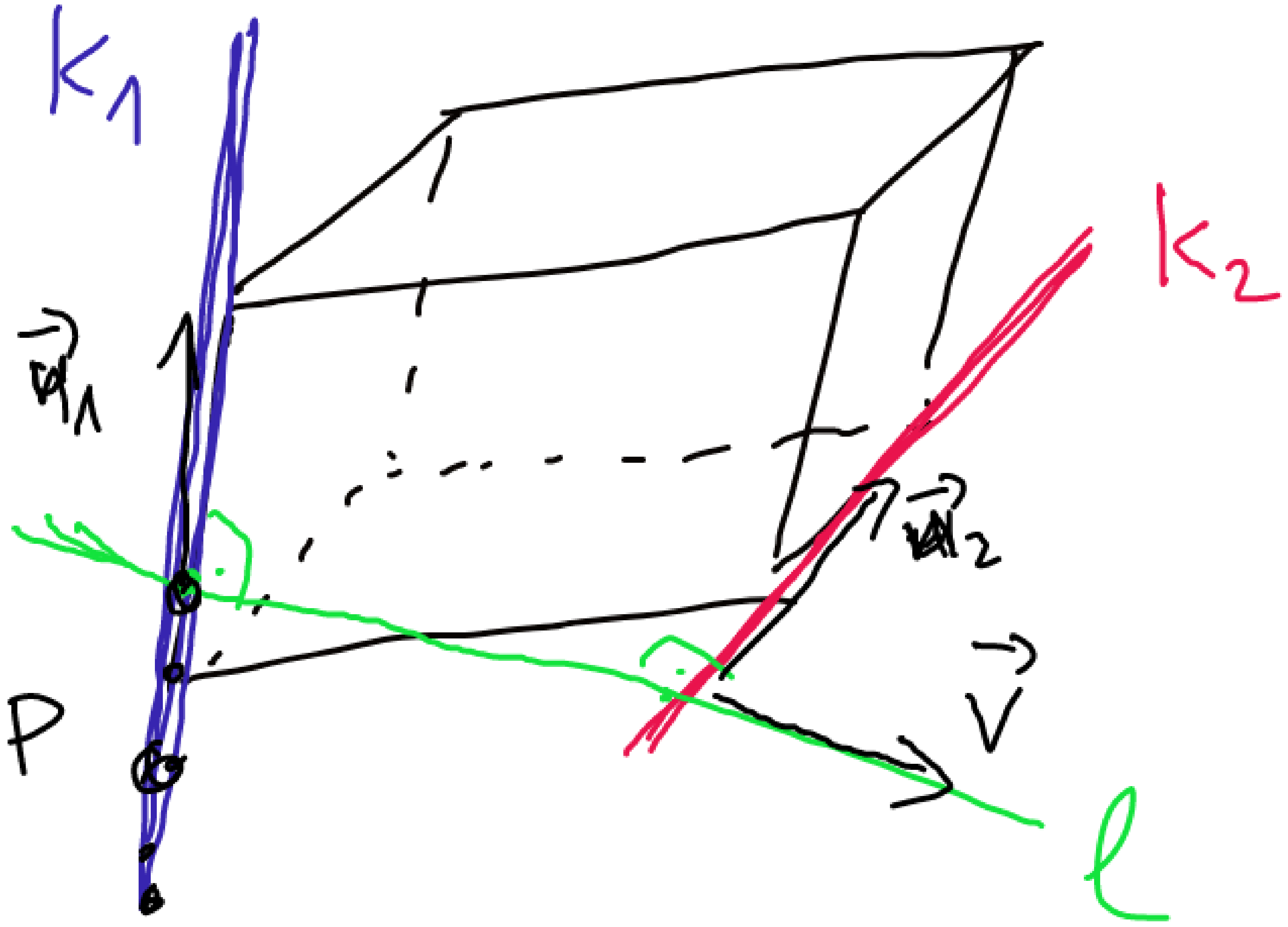
l' – obraz l przez symetrię względ. π
(albo: l' przechodzi przez P' i Q, gdzie P' – obraz P przez symetrię względ. π)

XXII

można wziąć

$$\cancel{\quad}\ \vec{u_1} \times \vec{u_2} = (-6, -6, -6)$$

$$\vec{v} = (1,1,1)$$



$\vec{u_1} = (-3, 1, 2)$

$\vec{u_2} = (0, 2, -2)$

punkt z k_1:

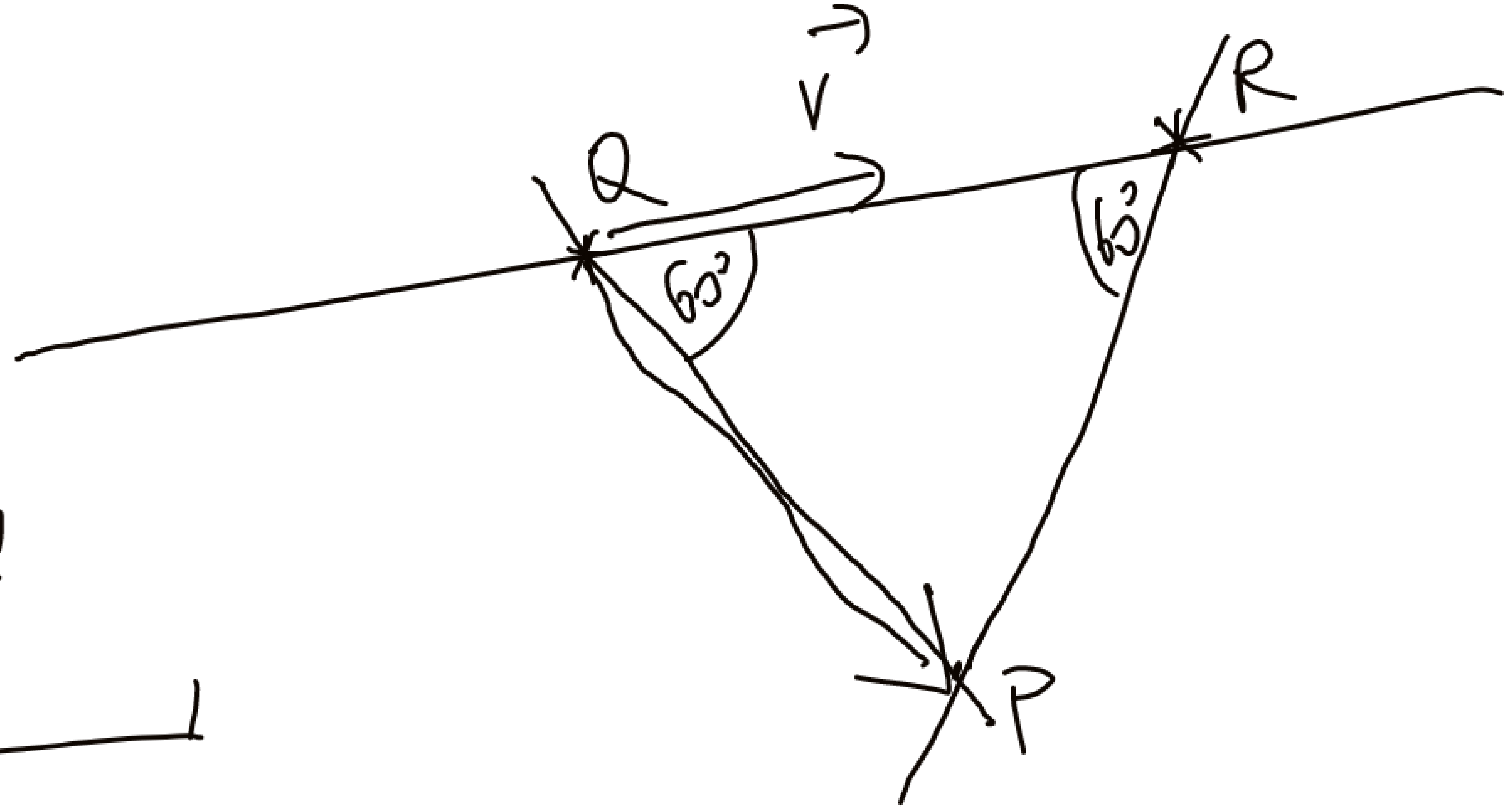
$$P = (-4-3t,\ t,\ 1+2t)$$

$$l:\ (x,y,z) = P + s\cdot\vec{v} = (-4-3t,\ t,\ 1+2t) + s\cdot(1,1,1) \quad \wedge\ s\in\mathbb{R}$$

Pytanie: dla jakiego t prosta l przecina k_2?

$$\begin{cases} (x,y,z) = (-4-3t,\ t,\ 1+2t) + (s,s,s) \\ \boxed{\dfrac{x-2}{0} = \dfrac{y}{2} = \dfrac{z-4}{-2}} \iff \begin{array}{l} x-2=0 \\ \dfrac{y}{2} = \dfrac{z-4}{-2} \end{array} \end{cases}$$

XXIII



$$\frac{x-3}{3} = \frac{y+2}{-1} = \frac{z-4}{2}$$

$$\vec{QP} \circ \vec{v} = |\vec{QP}| \cdot |\vec{v}| \cdot \cos \underbrace{\measuredangle(\vec{QP}, \vec{v})}_{=60^\circ \text{ lub } 120^\circ}$$

czyli

$$\left|\vec{QP} \circ \vec{v}\right| = \frac{1}{2} |\vec{QP}| \cdot |\vec{v}|$$

$$\cos\measuredangle = \frac{1}{2} \text{ lub } -\frac{1}{2}$$

XXV $l \subset \pi$, jeśli dwa różne punkty $P, Q \in l$ należą do π

XXVI