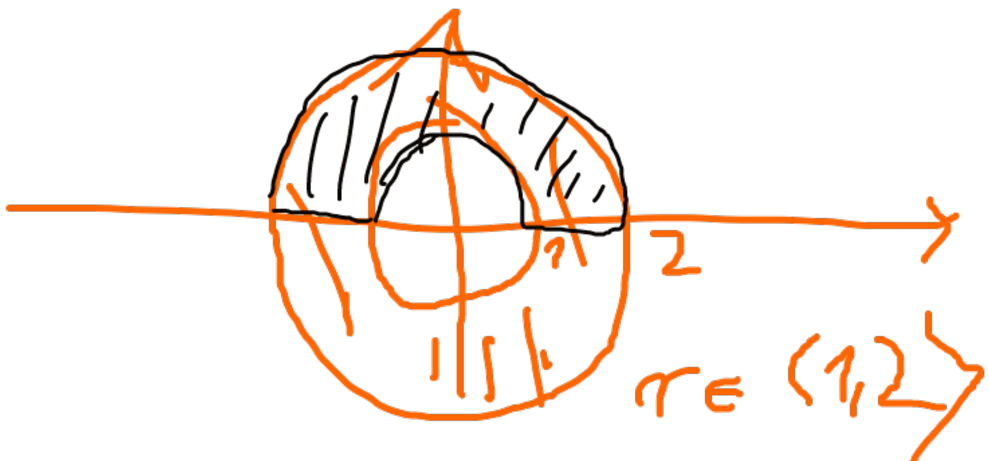
$$\iint_D \frac{\ln(x^2+y^2)}{x^2+y^2}\,dx\,dy$$

$$D=\{1\le x^2+y^2\le 4,\ y\ge 0\}$$

$$\begin{cases} x=r\cos\varphi \\ y=r\sin\varphi \end{cases}$$

$$dx\,dy = r\,dr\,d\varphi$$



$r\sin\varphi>0$

$\sin\varphi>0 \Rightarrow \varphi\in\langle 0,\pi\rangle$

$$\int_0^{\pi}\int_1^2 \frac{\ln r^2}{r^2}\,r\,dr\,d\varphi =$$

$$= \int_1^2 dr\ r\frac{\ln r^2}{r^2}\left[\varphi\right]_0^{\pi} =$$

$$= \int_1^2 dr\left(\frac{2\ln r}{r}\cdot\pi\right) = \left|\begin{matrix} t=\ln r \\ dt=\frac{1}{r}dr \end{matrix}\right| =$$

$$= \int_{\ln 1}^{\ln 2} 2\pi\, t\,dt = 2\pi\left[\frac{t^2}{2}\right]_{\ln 1}^{\ln 2} = 2\pi\left[\frac{(\ln 4)^2}{2}-\frac{(\ln 1)^2}{2}\right] =$$

$$= 2\pi\,\frac{\ln^2 2}{2}$$