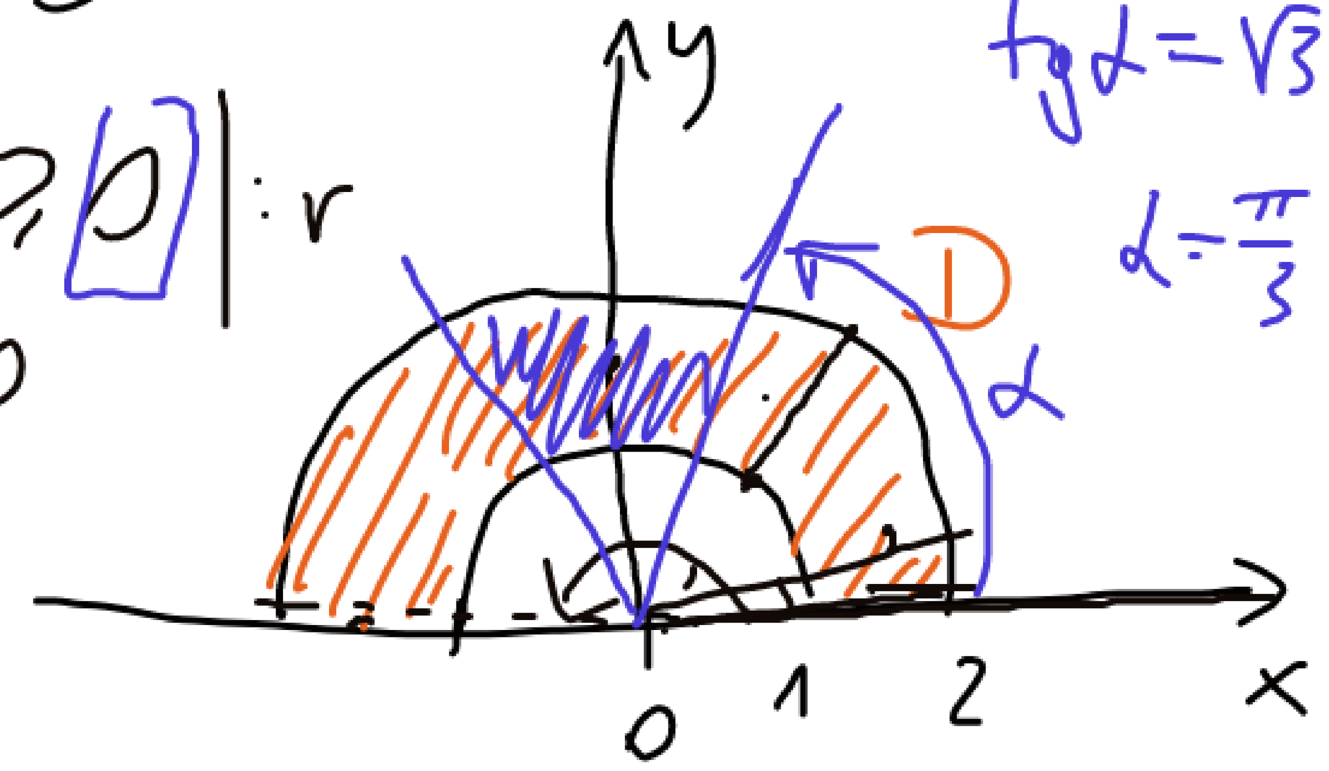
XIX/4

$$\iint\limits_D \sqrt{\frac{x^2+y^2}{1+x^2+y^2}}\,dx\,dy = (*) \qquad D=\{1\le x^2+y^2\le 4,\ y\ge \boxed{0}\}\ \sqrt{3}|x|$$

$\mathrm{tg}\,\alpha=\sqrt3$

$\alpha=\frac{\pi}{3}$



$$\left|\begin{array}{l} 1\le r^2\le 4,\ r\sin\varphi\ge\boxed{0}\ \Big|:r \\ r\in[1,2] \qquad \sin\varphi\ge 0 \\ \qquad \varphi\in[0,\pi] \end{array}\right.$$

$$\begin{cases} x=r\cos\varphi \\ y=r\sin\varphi \end{cases} \qquad \varphi\in\left[\cancel{0,\pi}\right] \quad \left[\frac{\pi}{3},\frac{2}{3}\pi\right], \quad r\in[1,2]$$

$$(*)=\int_0^{\pi} d\varphi \int_1^2 \sqrt{\frac{r^2}{1+r^2}}\; r\,dr = \int_0^{\pi} d\varphi \int_1^2 r\cdot\underbrace{\boxed{\frac{r}{\sqrt{1+r^2}}\,dr}}_{dt} = \left|\begin{array}{l} t=\sqrt{1+r^2} \\ dt=\frac{1}{\cancel{2}\sqrt{1+r^2}}\cdot \cancel{2}r\,dr \end{array}\right| =$$

$t^2=1+r^2$

$t^2-1=r^2$

$r=\sqrt{t^2-1}$

$$=\int_0^{\pi} d\varphi \int_{\sqrt2}^{\sqrt5} \sqrt{t^2-1}\,dt =$$

$$\int \sqrt{t^2-1}\,dt = \left|\begin{array}{l} t=\cosh x,\ x\ge 0 \\ dt=\sinh x\,dx \end{array}\right| = \int\sqrt{\cosh^2 x-1}\,\sinh x\,dx = \int \sinh^2 x\,dx =$$

$$\left.\begin{array}{l} \cosh x=\dfrac{e^x+e^{-x}}{2} \\ \sinh x=\dfrac{e^x-e^{-x}}{2} \end{array}\right\} + \qquad \cosh^2 x-\sinh^2 x=1$$

$$\boxed{\cosh^2 x - 1 = \sinh^2 x}$$

$$=\int\left(\frac{e^x-e^{-x}}{2}\right)^2 dx = \int \frac{e^{2x}-2+e^{-2x}}{2}\,dx = \frac12\left(\frac{e^{2x}}{2}-\frac{e^{-2x}}{2}\right)-x+C = \frac12\left(\frac{e^{2\ln(\ldots)}}{2}-\ldots\right.$$

$\cosh x+\sinh x = e^x$

$$x=\ln(\cosh x+\sinh x)=\ln\left(\cosh x+\sqrt{\cosh^2 x-1}\right)=\ln\left(t+\sqrt{t^2-1}\right)$$