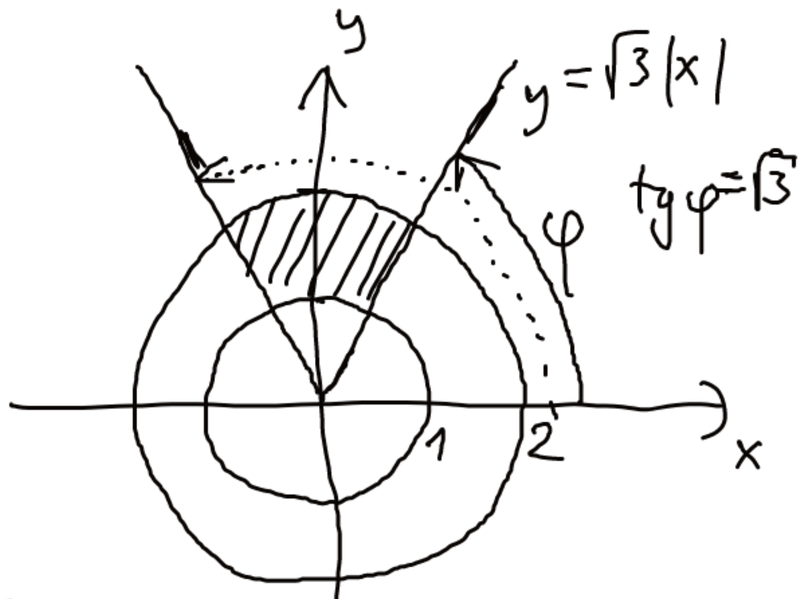
$$\iint_D \sqrt{\frac{x^2+y^2}{1+x^2+y^2}}\,dx\,dy$$

$$D = \left\{1 \le x^2+y^2 \le 4,\ y \ge \sqrt{3}|x|\right\}$$



$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$$

$$dx\,dy = r\,dr\,d\varphi$$

$r \in [1,2]$

$\varphi \in \left[\frac{\pi}{3}, \frac{2}{3}\pi\right]$

$$\int_{\frac{\pi}{3}}^{\frac{2}{3}\pi} d\alpha \int_1^2 \sqrt{\frac{r^2}{1+r^2}}\, r\,dr$$

$$\int \sqrt{\frac{r^2}{1+r^2}}\, r\,dr = \left| \begin{array}{l} t = r^2 \\ dt = 2r\,dr \\ \frac{1}{2}dt = r\,dr \end{array} \right| = \frac{1}{2}\int \sqrt{\frac{t}{1+t}}\,dt$$

$$u = \sqrt{\frac{t}{1+t}}$$

$$u^2 = \frac{t}{1+t} = 1 - \frac{1}{1+t} \Rightarrow \frac{1}{1+t} = 1-u^2$$

$$2u\,du = \frac{1}{(1+t)^2}\,dt = (1-u^2)^2\,dt$$

$$\frac{2u}{(1-u^2)^2}\,du = dt$$

$$= \frac{1}{2}\int u\,\frac{2u}{(1-u^2)^2}\,du$$