19/8

$$\iint_D \operatorname{arctg}\frac{y}{x}\,dx\,dy =$$

$D = \{x^2+\underbrace{y^2+4y}<0,\ x\le 0,\ y\le x\sqrt{3}\}$

$$= \int_{4/3\pi}^{\frac{3}{2}\pi} d\varphi \int_0^{-4\sin\varphi} \operatorname{arctg}\overbrace{\frac{r\sin\varphi}{r\cos\varphi}}^{=\operatorname{tg}\varphi}\; r\; dr =$$

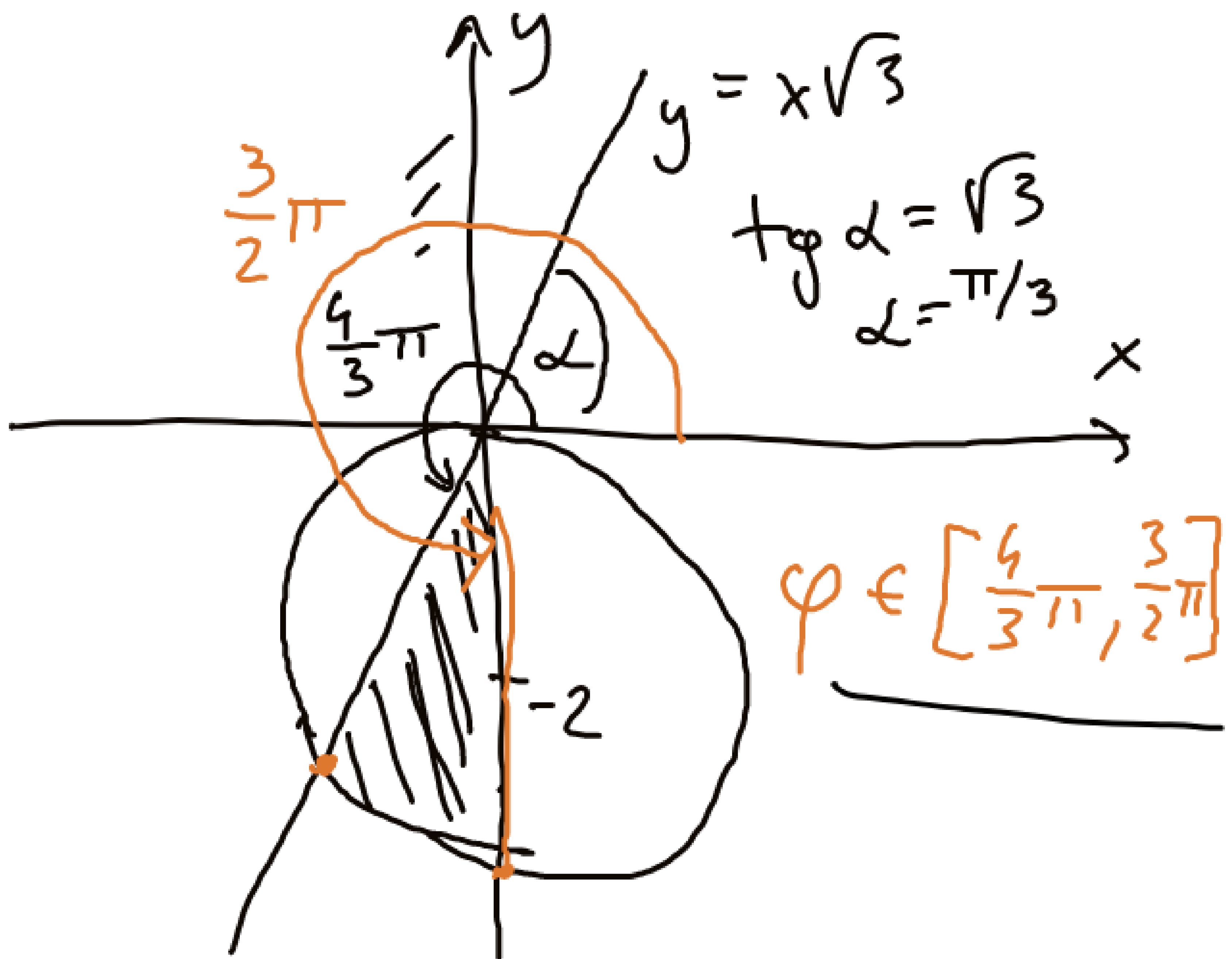
$x^2+(y+2)^2-4<0$

$x^2+(y+2)^2<2^2$

$$= \int_{\frac{3}{2}\pi}^{\frac{4}{3}\pi} d\varphi \int_0^{-4\sin\varphi} r\ \operatorname{arctg}(\operatorname{tg}\varphi)\ dr =$$

$$= \int_{3/2\pi}^{4/3\pi} \operatorname{arctg}(\operatorname{tg}\varphi)\ \frac{r^2}{2}\Big|_{r=0}^{r=-4\sin\varphi} d\varphi$$

$$= \int_{3/2\pi}^{4/3\pi} \operatorname{arctg}(\operatorname{tg}\varphi)\ \frac{4^2\sin^2\varphi}{2}\ d\varphi = (*)$$



Pytanie $\operatorname{arctg}(\operatorname{tg}\varphi) = ?$

I

$\operatorname{arctg} y = x \Leftrightarrow \operatorname{tg} x = y,\ x\in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\operatorname{tg} x = \operatorname{tg}\varphi \quad \varphi\in[4/3\pi, 3/2\pi]$

$\quad\quad = \operatorname{tg}(\underbrace{\varphi-\pi})$

$x = \varphi - \pi \in \left[\frac{1}{3}\pi, \frac{1}{2}\pi\right]$

$\operatorname{arctg}(\operatorname{tg}\varphi) = \varphi - \pi$

$$(*) = \int_{3/2\pi}^{4/3\pi} (\varphi-\pi)\cdot 8\sin^2\varphi\, d\varphi = \ldots$$

$x^2+y^2+4y<0 \qquad \begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases} \quad r\ge 0$

$r^2+4r\sin\varphi<0$

$r^2 < -4r\sin\varphi \quad \Big|: r>0$

$0 < r < -4\sin\varphi \quad (\varphi\in(\pi, 2\pi))$

z (2) ; (3) $\varphi\in[4/3\pi, 3/2\pi]$