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$$x^2 + y^2 \leq 4$$

$$0 \leq r^2 \leq 4$$

$$0 \leq r \leq 2$$

$$r \cos \alpha \leq 0$$

$$\cos \alpha \leq 0$$

$$\alpha \in \left[\frac{\pi}{2}, \frac{3\pi}{2} \right]$$

$$r \sin \alpha > 1$$

CO DANE], NIE WIEM

1...

bo $r \geq 2$

$$\sin \alpha \geq \frac{1}{r} \geq \frac{1}{2}$$

o, oznaczcie
trójkąt



$$\alpha \in \left[\frac{\pi}{2}, \frac{5\pi}{6} \right]$$

$$\alpha \in \left[\frac{\pi}{6}, \frac{5\pi}{6} \right]$$

dodałowa $r \geq \frac{1}{\sin \alpha}$

ups



$$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} d\alpha \int_{\frac{1}{\sin \alpha}}^2 \frac{1}{(r^2)^2} r dr = (\dots)$$

$$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \frac{1}{\sin \alpha} d\alpha$$

$$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \frac{1}{\sin \alpha} d\alpha = \left. \begin{array}{l} t = r^2 \\ dt = 2r dr \\ \frac{dt}{2} = r dr \end{array} \right| = \int_{\frac{1}{\sin^2 \alpha}}^4 \frac{1}{t^2} \cdot \frac{dt}{2} =$$

$$= \frac{1}{2} \left[-t^{-1} \right]_{\frac{1}{\sin^2 \alpha}}^4 = \frac{1}{2} \left[-\frac{1}{4} + \sin^2 \alpha \right] = -\frac{1}{8} + \frac{\sin^2 \alpha}{2}$$

$$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \left(-\frac{1}{8} + \frac{\sin^2 \alpha}{2} \right) d\alpha = -\frac{1}{8} \cdot \frac{5\pi}{6} + \frac{1}{8} \cdot \frac{\pi}{2} + \frac{1}{2} \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \sin^2 \alpha d\alpha =$$

$$= -\frac{5\pi}{48} + \frac{\pi}{16} + \frac{1}{2} \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \frac{1 - \cos 2\alpha}{2} d\alpha = \dots$$