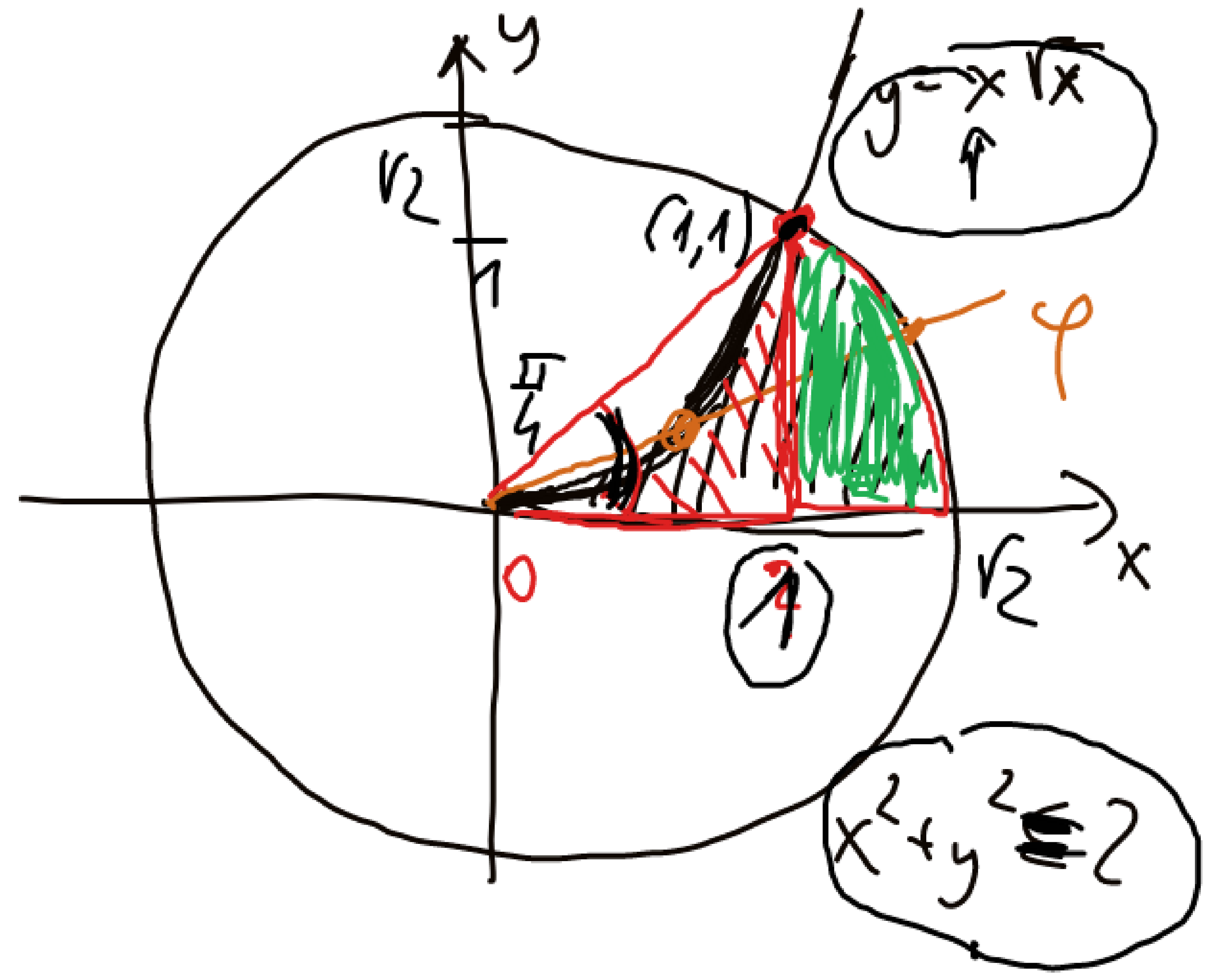
$x^2+y^2\le 2$, $\quad 0\le y\le x\sqrt{x}=x^{3/2}$



$\begin{cases} x=r\cos\varphi \\ y=r\sin\varphi \end{cases}$ $\qquad \varphi\in[0,2\pi]$, $\quad \underbrace{r\ge 0}$

$r^2\le 2 \qquad 0\le r\sin\varphi\le (r\cos\varphi)^{3/2}$

$\underbrace{r\in[0,\sqrt{2}]}$

$\sin\varphi\ge 0$ $\qquad$ żeby miało sens: $\cos\varphi\ge 0$ $\quad$ zat. $\cos\varphi>0$

$\underbrace{\varphi\in[0,\pi]} \longrightarrow \varphi\in[0,\frac{\pi}{2})$

$0\le r\sin\varphi\le r^{3/2}\cos^{3/2}\varphi \quad \Big|:r>0$

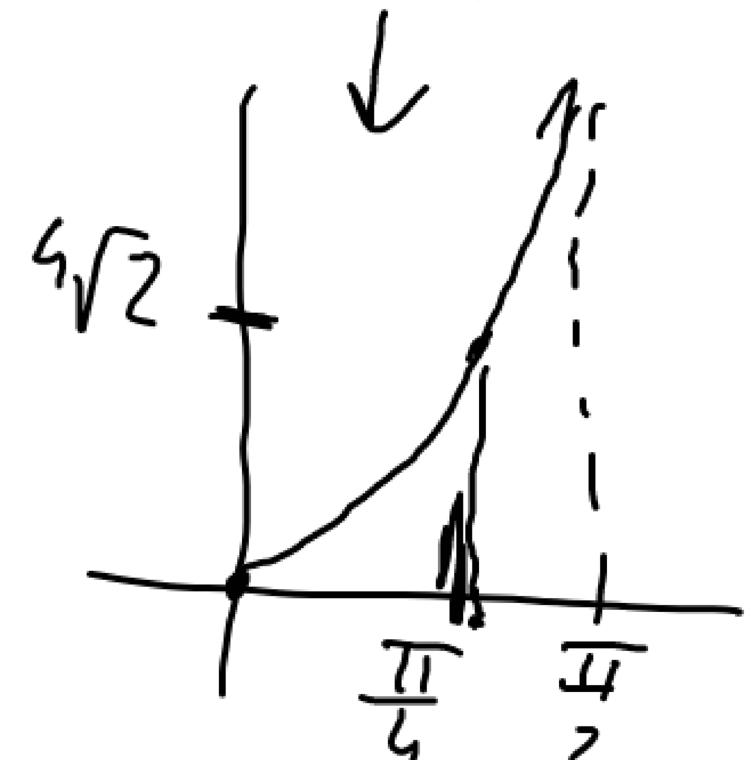
$0\le \sin\varphi\le \sqrt{r}\cos^{3/2}\varphi \quad \Big|:\cos^{3/2}\varphi$

$0\le \underbrace{\frac{\sin\varphi}{\cos^{3/2}\varphi}\le\sqrt{r}}\le\sqrt[4]{2} \longrightarrow \frac{\sin^2\varphi}{\cos^3\varphi}\le r$

w szczególności

$\frac{\sin\varphi}{\cos^{3/2}\varphi}\le\sqrt[4]{2}$ $\qquad$ $\sin\varphi\nearrow$, $\cos\varphi\searrow$ na $[0,\frac{\pi}{2}]$ $\qquad$ $\frac{\sin\varphi}{\cos^{3/2}\varphi}\nearrow$

$\varphi=\frac{\pi}{4}$ $\qquad \frac{\sin\frac{\pi}{4}}{\cos^{3/2}\frac{\pi}{4}}=\frac{\frac{\sqrt2}{2}}{\left(\frac{\sqrt2}{2}\right)^{3/2}}=\frac{1}{\left(\frac{\sqrt2}{2}\right)^{1/2}}=\left(\frac{2}{\sqrt2}\right)^{1/2}=\sqrt[4]{2}$



$\begin{cases} \varphi\in[0,\frac{\pi}{4}] \\ \frac{\sin^2\varphi}{\cos^3\varphi}\le r\le\sqrt{2} \end{cases}$

$$P=\int_0^{\frac{\pi}{4}}d\varphi\int_{\frac{\sin^2\varphi}{\cos^3\varphi}}^{\sqrt2} r\,dr=\int_0^{\frac{\pi}{4}}\left(\frac{\sqrt2^2}{2}-\frac{\sin^4\varphi}{2\cos^6\varphi}\right)d\varphi$$