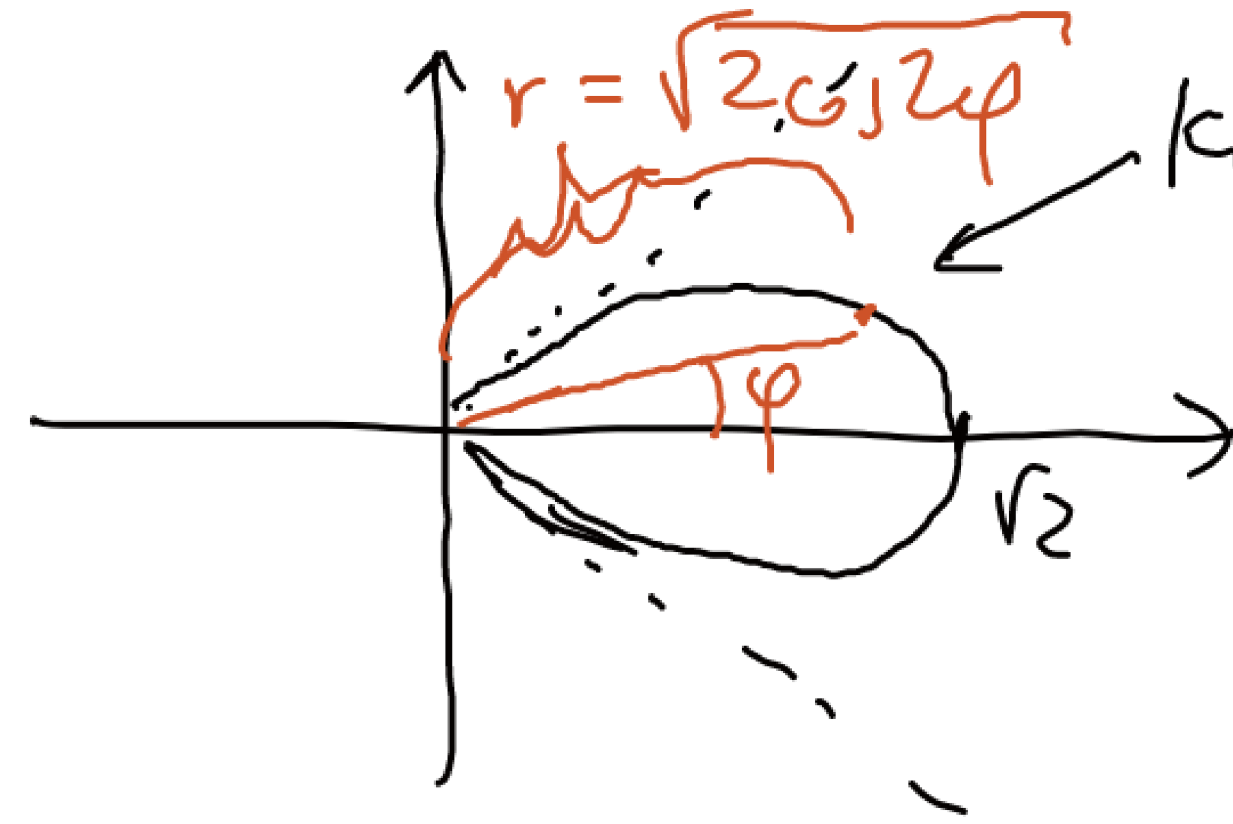
20/2

$$(x^2+y^2)^2 = 2(x^2-y^2)$$




Krzywa $r^2=2\cos 2\varphi$ (mniej więcej)

podstawiamy $\begin{cases} x=r\cos\varphi \\ y=r\sin\varphi \end{cases}$ . . .

$$(r^2)^2 = 2(r^2\cos^2\varphi - r^2\sin^2\varphi)$$

$$r^4 = 2r^2\cos 2\varphi$$

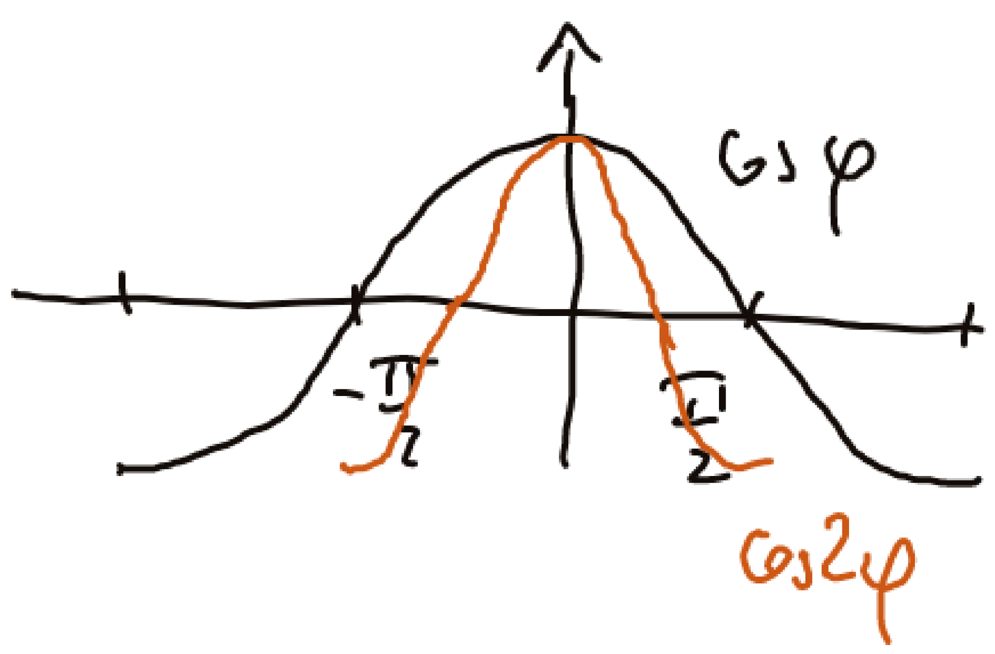
$$0 \leq r^2 = 2\cos 2\varphi$$

np. dla $\varphi=0$ : $r^2=2$ $\quad r=\sqrt{2}$

$\varphi=\pm\frac{\pi}{4}$ $\quad r^2=2\cos(\pm\frac{\pi}{2})=0$

gdy $\varphi$ zwiększa się od $0$ do $\frac{\pi}{2}$, to $\cos 2\varphi$ zmniejsza się od $1$ do $0$

jeśli $\varphi\in[-\pi,\pi]$, to $\cos 2\varphi \geq 0$ na $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$




Zbiór ograniczony tą krzywą: $\varphi\in\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$, $r\in\left[0, \sqrt{2\cos 2\varphi}\right]$

$$\text{Pole} = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} d\varphi \int_0^{\sqrt{2\cos 2\varphi}} r\, dr = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left.\frac{r^2}{2}\right|_{r=0}^{r=\sqrt{2\cos 2\varphi}} d\varphi =$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\cancel{2}\cos 2\varphi}{\cancel{2}}\, d\varphi = \left.\frac{\sin 2\varphi}{2}\right|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{1}{2}-\left(-\frac{1}{2}\right) = 1.$$