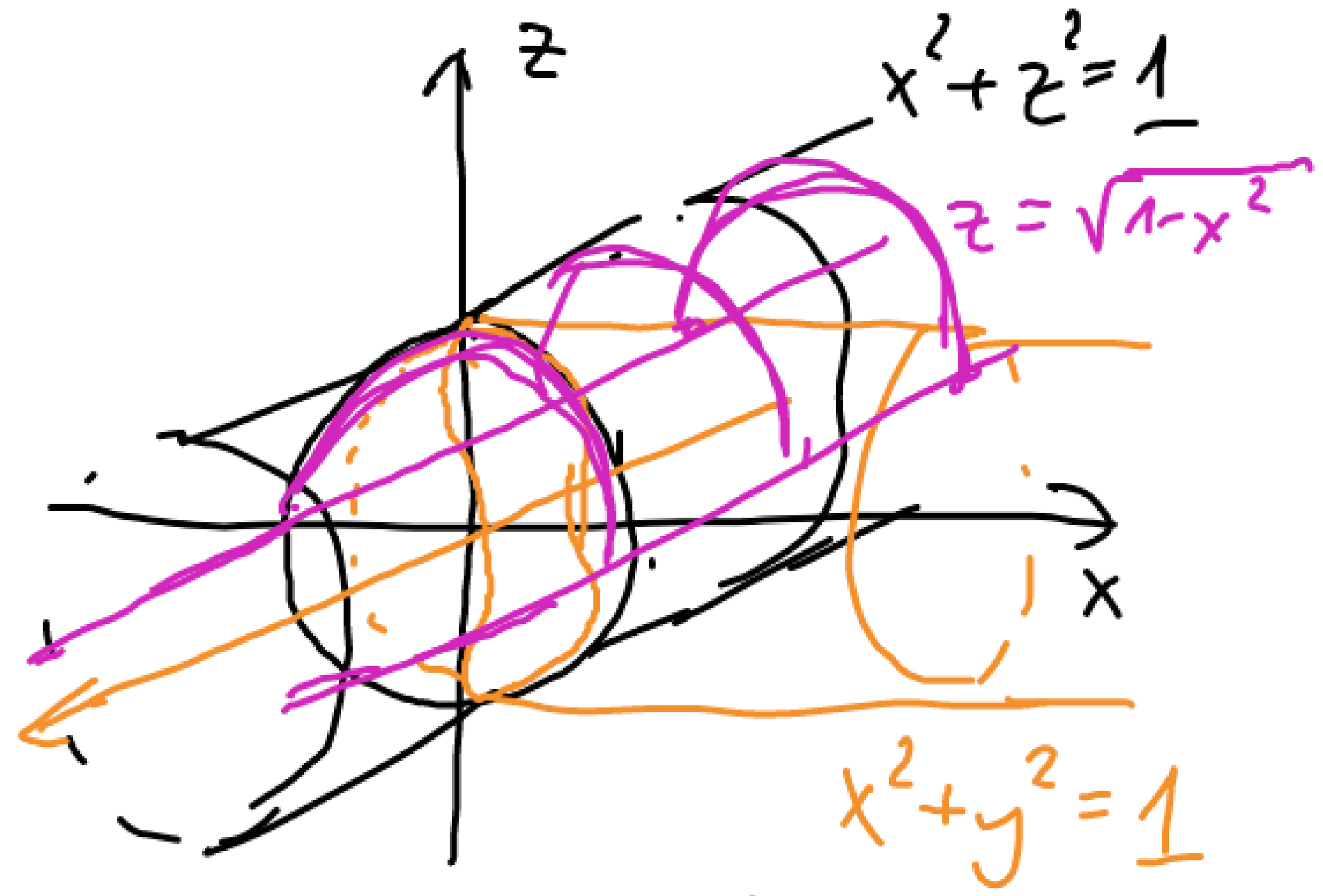
XXII/2

$x^2+y^2=1$ $\qquad x^2+z^2=1$

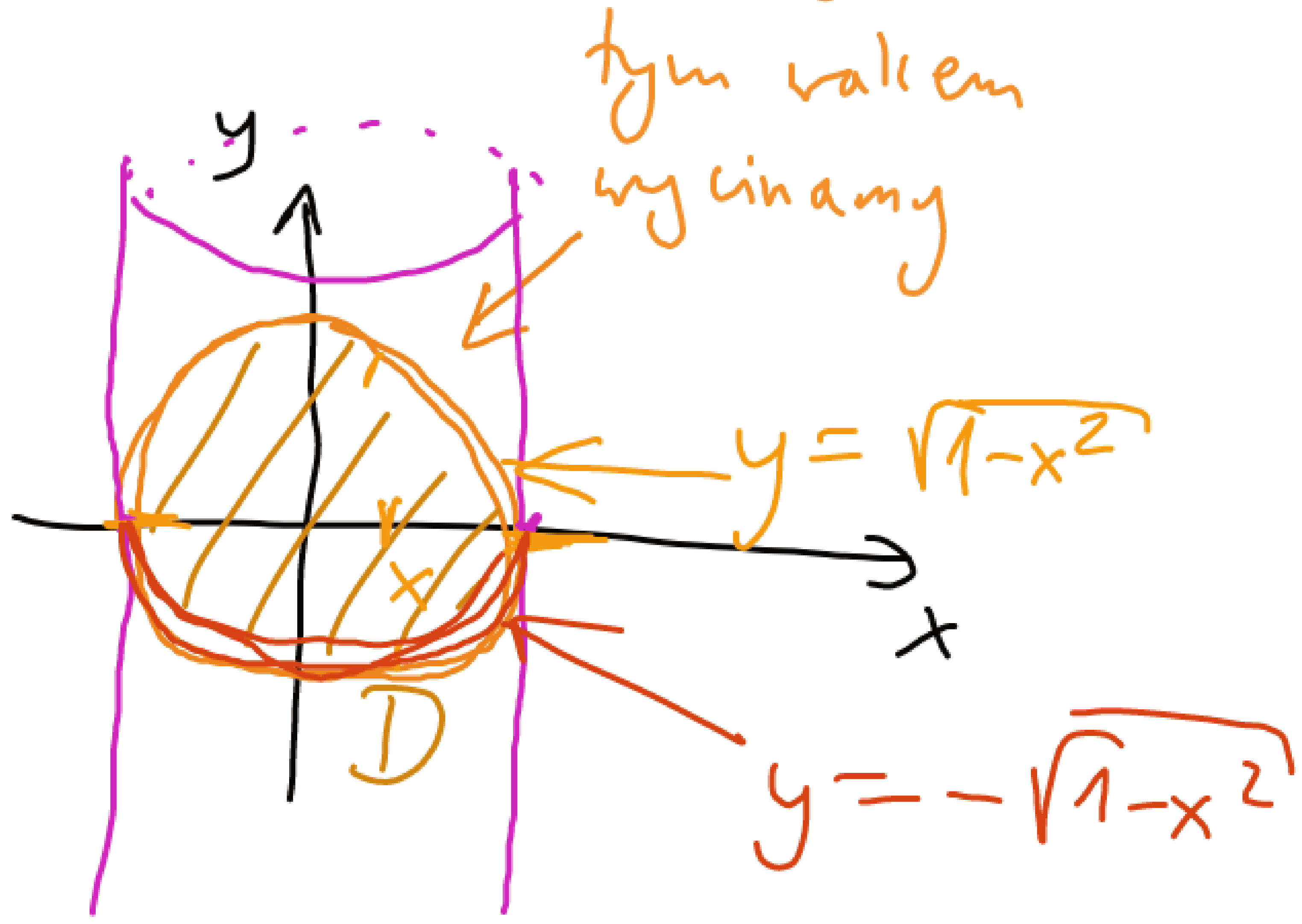
$$z^2=1-x^2$$

$$z=-\sqrt{1-x^2} \lor z=\sqrt{1-x^2}$$



policzmy pole połowy danej powierzchni, czyli pole wykresu f-cji $z(x,y)=\sqrt{1-x^2}$ dla $(x,y)\in D$

$x\in[-1,1]$, $y\in\mathbb{R}$



$$P=\iint_D \sqrt{1+(z_x)^2+(z_y)^2}\,dx\,dy=$$

$$=\iint_D \sqrt{1+\left(\frac{1}{\cancel{2}\sqrt{1-x^2}}\cdot(-\cancel{2}x)\right)^2+0^2}\,dx\,dy=\iint_D \sqrt{1+\frac{x^2}{1-x^2}}\,dx\,dy=$$

$$=\iint_D \sqrt{\frac{1-x^2+x^2}{1-x^2}}\,dx\,dy=\iint_D \frac{1}{\sqrt{1-x^2}}\,dx\,dy=\int_{-1}^{1}dx\underbrace{\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\frac{1}{\sqrt{1-x^2}}\,dy}=$$

$$=\int_{-1}^{1}(y)\Big|_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}}\cdot\frac{1}{\sqrt{1-x^2}}\,dx=\int_{-1}^{1}2\sqrt{1-x^2}\cdot\frac{1}{\sqrt{1-x^2}}\,dx=4.$$

Odp. 8.