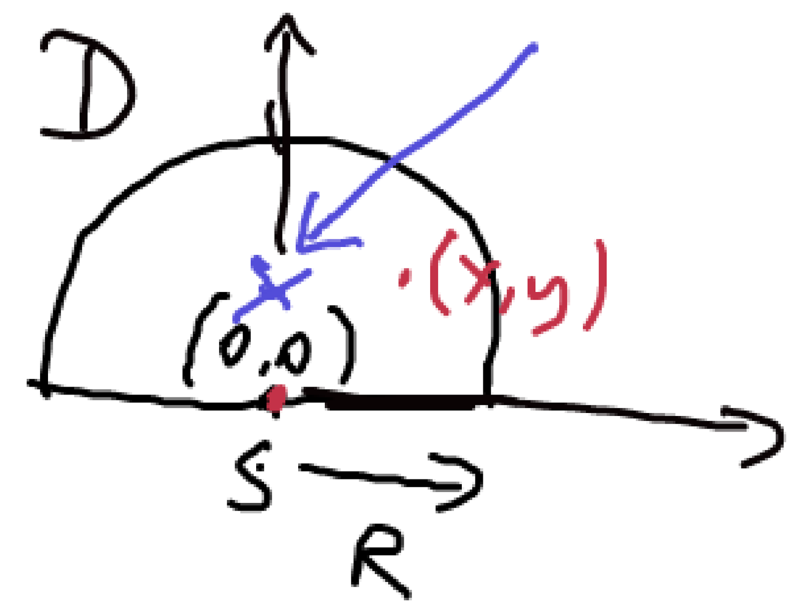
XXVI/1

$$D: \begin{cases} x^2+y^2 \le R^2 \\ y \ge 0 \end{cases}$$



$$M = \iint_D \underbrace{\gamma\sqrt{x^2+y^2}}\,dx\,dy =$$

$r \in [0,R]$

$\varphi \in [0,\pi]$

$$= \int_0^\pi d\varphi \int_0^R \gamma\sqrt{r^2}\cdot r\,dr = \gamma\int_0^\pi d\varphi \int_0^R r^2\,dr = \gamma\int_0^\pi d\varphi\, \frac{r^3}{3}\Big|_0^R =$$

$$= \gamma\frac{R^3}{3}\,\varphi\Big|_0^\pi = \gamma\frac{\pi R^3}{3}$$

$$M_x = \iint_D x\,\gamma\sqrt{x^2+y^2}\,dx\,dy = \ldots = 0$$

$$M_y = \iint_D y\,\gamma\sqrt{x^2+y^2}\,dx\,dy = \int_0^\pi d\varphi \int_0^R r\sin\varphi\cdot\gamma\sqrt{r^2}\,r\,dr =$$

$$= \int_0^\pi d\varphi\; \gamma\sin\varphi\, \frac{r^4}{4}\Big|_{r=0}^{r=R} = \int_0^\pi \sin\varphi\,d\varphi\cdot\gamma\frac{R^4}{4} =$$

$$= -\cos\varphi\Big|_0^\pi\,\gamma\frac{R^4}{4} = 2\gamma\frac{R^4}{4}$$

$$\text{środek masy} = \left(\frac{M_x}{M}, \frac{M_y}{M}\right) = \left(0, \frac{2\cancel{\gamma}\frac{R^4}{4}}{\cancel{\gamma}\frac{\pi R^3}{3}}\right) = \left(0, \frac{3}{2\pi}R\right)$$

$$\rho(x,y) = \text{gęstość masy}_{\,w\,(x,y)} =$$

$$= \gamma\cdot \text{odl}((x,y), S)$$

$$= \gamma\sqrt{x^2+y^2}$$

$$M = \iint_D \rho(x,y)\,dx\,dy$$

masa

$$M_x = \iint_D x\,\rho(x,y)\,dx\,dy$$

$$M_y = \iint_D y\,\rho(x,y)\,dx\,dy$$

$$\to \left(\frac{M_x}{M}, \frac{M_y}{M}\right) - \text{środek masy}$$